Monetary Transmission and the Search for Liquidity

Victor E. Li, Villanova University

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Abstract: This paper evaluates the implications of search and matching frictions in the financial market for the transmission of monetary policy. Borrowers and lenders participate in a decentralized loan market for the purpose of establishing long-term credit relationships and the provision of loanable funds to productive firms. Locating credit relationships is costly in terms of time and real resources and the interest rate is negotiated via a bargaining mechanism. This structure is incorporated into an otherwise standard monetary business cycle framework and used to study how such frictions in the credit market contribute to explaining the contemporaneous impact and propagation of monetary growth shocks and inflation. It is found that while anticipated inflation negatively impacts real activity it can also increases loan market participation and the inflow of newly established credit relationships. It is shown that while bargaining and costly search mitigates the traditional inflation tax effect of monetary injections, the existence of long term lending relations tend to dampen the immediate liquidity effects. The model also indicates that there may not necessarily exist a negative correlation between credit market tightness and aggregate activity. Furthermore, search frictions provide a potentially important mechanism for explaining the persistence of monetary shocks, an issue that has been problematic in limited participation models of the transmission mechanism.

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Correspondence: Victor Li, Department of Economics, Villanova University, 800 Lancaster Ave, Villanova, PA, Victor.Li@villanova.edu.
1 Introduction

There is little doubt that credit market frictions play an important role in the transmission of monetary policy to real aggregate economic variables. Most of the literature advocating this credit channel captures these market imperfections by introducing asymmetric information which give rise to moral hazard in lending relationships. Yet only very recently has the difficulty faced by entrepreneurs in locating and establishing credit relationships, as documented by Blanchflower and Oswald (1998), been taken seriously as another potentially important source of friction in credit markets. As such, relatively little work has been done evaluating the consequences of financial markets which explicitly capture the idea that locating credit opportunities is not a costless and instantaneous process but requires borrowers and lenders to expend time and real resources. These credit market search frictions, which can arise from incomplete information regarding lending opportunities, implies that in any given period there will be a coexistence of idle loanable funds along with an unfulfilled demand for these funds. They also provide value in establishing long term credit relationships.

This paper constructs a dynamic general equilibrium monetary model which incorporates search frictions into the financial market and investigates its implications for the monetary transmission mechanism. Our model features a decentralized financial market where households (or lenders), who are faced with an intertemporal savings decision, must carry deposits while seeking borrowers. Firms (or borrowers), who own the production technology, invest costly effort in the search for loanable funds to finance the factors of production. Once a credit relationship is established the nominal interest rate characterizing the terms of the loan is negotiated by a Nash bargaining solution that divides the total match surplus. This credit relationship is then carried over into the future where there is an exogenous probability that the relationship will dissolve. Hence, our financial market differs from the frictionless case in two respects. First, there is costly search and matching which lead to loan creation. Secondly, there exists long-term lending relationships. To focus specifically on the impact of the evolving stock of lending relationships we abstract from capital accumulation and assume that labor is the only production factor.

We embed this financial structure into a conventional monetary framework that features a limited participation mechanism to generate the liquidity effects of monetary injections.

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1The pioneering work of Bernanke and Gertler (1989) and quantitative extensions by Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999), and Carlstrom and Fuerst (2000) explicitly demonstrate that such agency costs issues endogenously generate financial constraints which can be amplified by the effect of aggregate shocks on borrower net worth. Kiyotaki and Moore (1997) study the interaction of collateralized debt and asset prices with these agency cost.
In particular, households must finance consumption purchases with money balances net of the funds that are either tied to existing credit relationships or used in the search of new relationships. The monetary authority also participates in the decentralized financial market by making additional cash transfers available as loanable funds. Having these funds available after the household chooses its deposit decision provides the limited participation mechanism where monetary transfers lead to a lower nominal interest rate.

The quantitative implications of this framework to changes in steady state money growth as well as persistent monetary shocks are evaluated and compared to the "frictionless" limited participated model.\textsuperscript{2} The results can be summarized by the following. We find that steady money growth and inflation can increase loan market participation on the part of borrowers and, for a sufficiently low separation rate, new loan creation. This loan market participation effect can be interpreted as a "shoe leather" costs of inflation incurred by unmatched borrowers in the loan market. At the same time, inflation increases credit market tightness, contracts the total real quantity of lending relationships and overall real activity. Hence, this result is generally consistent with the empirical evidence that credit and banking activity expands with higher inflation while the actual real quantity of loans contract.\textsuperscript{3}

There are several notable findings for the stochastic equilibrium. First, all else being equal, costly search and matching mitigates anticipated inflation effects and amplifies liquidity effects relative to the frictionless case. The reason for this can be traced to the impact of the money injection on the bargaining position of borrowers and lenders. For the lender, anticipated inflation increases the opportunity cost of searching with idle funds and hence the value of matching with a borrower. This places downward pressure on the nominal interest rate and partially offsets the increase associated with traditional anticipated inflation effects. For the borrower, the additional liquidity initially lowers credit market tightness, reduces their match surplus, and pushes the nominal interest rate down. An interesting corollary to this finding is that equilibrium credit market tightness (from the borrower’s perspective) can increase in the period of the liquidity effect while declining sharply afterward. Here the additional liquidity increases borrower search effort by more than the additional loanable funds. Hence, a tighter credit market can coexist with increased real economic activity.

Second, the existence of long-term lending relationships amplifies anticipated inflation effects. If a positive monetary innovation exhibits persistence then lenders will withdraw deposits and decrease loan market participation in the period after the shock. The consequence

\textsuperscript{2}We use the term "frictionless" to describe a basic cash-in-advance or limited participation framework with no financial market search frictions. Technically, of course, the limited participation constraint is itself a form of friction.

\textsuperscript{3}See Aiyagari, Braun, and Eckstein (1998), English (1999), and Boyd and Champ (2006).
of this is a deterioration of the future stock of credit relationships which take time to rebuild. This increases the marginal value of future credit relationships for borrowers and their current search effort, leads to a tighter credit market, and increases their match value. Hence the implied increase in the nominal rate reinforces the higher rates associated with the anticipated inflation effect.

Finally, it is shown that a sufficiently high search costs or low bargaining share for borrowers will imply that monetary shocks can have persistent effects on the nominal interest rate and real economic activity. In particular, the nominal interest rate is shown to be below steady state for several periods while aggregates such as labor, consumption, and output will display the hump-shaped responses observed in the data. This result stems from the interaction between the decline in credit market tightness in the period after the shock and the borrower’s bargaining position. When search costs are high the marginal negative impact from additional easing of the credit market on borrowers’ match surplus in the period following the shock is much higher. This places continued downward pressure on the nominal interest rate leading to persistence. Obtaining this result is particularly problematic in limited participation models without ex ante portfolio adjustment costs as household deposit typically decrease in the period after the shock to undo the additional cash injection.\footnote{See Christiano and Eichenbaum (1992)}

\section{1.1 Related Literature}

While there is an extensive literature applying search theory in the labor market to macroeconomic models, spurred on by the contributions of Mortensen and Pissarides (1994), Merz (1995), and Andolfatto (1996), there is a much smaller but growing body of work investigating the aggregate implications of credit market search. Craig and Haubrich (2006) complement the empirical work of Dell’Arriccia and Garibaldi (2005) and compile evidence regarding gross credit flows via loan creation and destruction and the entry and exit of banks. They further demonstrate that a simple search theoretic model of lending may capture these flows evident in the data. Den Haan, Ramey, and Watson (2003) combine matching frictions with moral hazard and allocative frictions to study the propagation of real aggregate shocks. Becsi, Li, and Wang (2005,2008) study interest rate determination and the allocation of credit and in search theoretic models featuring heterogenous borrowers and informational-based frictions caused by moral hazard, respectively. Wasmer and Weil (2004) investigate the linkages between labor market and financial market search in a stationary environment while Nicoletti and Pierrard (2006) and Kurmann and Petrosky-Nadeau (2007) extend the analysis to a quantitative real business cycle model.
The literature on limited participation models of the monetary transmission mechanism stems from the theoretical work of Lucas (1990) and Fuerst (1992) and the quantitative implementation by Christiano (1991) and Christiano and Eichenbaum (1992). Some extensions of these models attempting to explain the persistence of monetary policy shocks include Cook (1999), who consider an externality by which past lending activity lowers intermediation costs, Andolfatto and Gomme (2003), who assume incomplete information regarding the monetary policy regime, and Bohacek and Mendizabal (2007), where banks exchange bonds for reserves and allocate liquidity between high and low productive lenders.

The only other study which integrates financial market search into a limited participation monetary framework is Hendry and Moran (2004). Similar to this paper, there is a decentralized loan market where borrowers and lenders meet according to a matching technology and interest rates are determined via a bargaining solution. However, their mechanism differs in that liquidity effects are propagated by an intermediation spread between loans and deposits and entrepreneurs who operate decreasing returns to scale production technologies.

1.2 Paper Structure

The paper proceeds as follows. Section 2 constructs the basic framework, characterizes the efficiency and market equilibrium conditions, and studies some properties of the steady state. Section 3 proceeds with the quantitative analysis of both the steady state and the stochastic equilibrium. It evaluates the contributions of costly search and long-term lending relationships relative to the base line limited participation model in the absence of credit market frictions. Finally Section 4 contains concluding remarks.

2 The Model

Time is discrete and the economy is populated with many identical and infinitely lived household and firms. The description of the competitive labor and goods markets are fairly standard. The decentralized financial market which channels loanable funds from households and firms is described in the subsection below. Each household consists of a continuum of members indexed over the unit interval. In each period $t$ the household decides the division of work effort and leisure, where the set of those allocated to work having measure $n_t \in (0, 1)$ and those to leisure $(1 - n_t)$. Those supplying labor receive a nominal wage payment $W_t$. To finance consumption purchases household carry nominal money balances $M_t$ and deposits a portion into the decentralized loan market that earns nominal interest rate $R_t$. Household preferences over consumption $c_t$ and leisure $L = (1 - n_t)$ is described by $u(c_t, 1 - n_t)$ which is
increasing and concave in both arguments and the typical Inada conditions are satisfied.

Each firm operates a constant returns to scale technology and produces output with an exogenous level of capital stock (normalized to one) and labor: \( Y_t = f(n_t) \). Keeping the stock of capital fixed simplifies the analysis and allows us to focus on the dynamics arising from matching in the financial market. The nominal wage payment \( W_t \) must be financed by funds borrowed from the financial market at nominal interest rate \( R_t \).

2.1 Financial Market Matching

The decentralized loan market features an environment where households and firms must engage in costly search for the establishment of credit relationships. As in Becsi, Li, and Wang (2005) there is no explicit financial intermediary so we will use the terms households/lenders and firms/borrowers interchangeably. To be directly comparable to existing quantitative limited participation models without credit market frictions we parallel the labor market model of Merz (1995) and imagine lenders and borrowers belonging to large risk-sharing households/firms. Such a structure allows us to circumvent wealth distributions which arise due to the random nature of the search process. Households participate in the loanable funds market by allocating its members to the financial market. These "loan agents," are either inactive, matched and supplying loanable funds, or unmatched and searching for lending opportunities.

Each unmatched loan agent carries \( P_t \) nominal dollars, funded by household deposits, into the financial market. Denote the measure of matched and unmatched loan agents as \( e^b_H \) and \( e^l_t \), respectively. Each dollar of funds provided to matched borrowers pays a nominal interest rate return \( R_t \). Note that the labor and financial market allocation decisions are independent so members supplying work effort or enjoying leisure are not precluded from participating in the financial market and vice versa. Each firm consists of a continuum of borrowers who are either inactive, matched and receiving loanable funds, or unmatched and searching for financing opportunities. We will denote the measure of matched and unmatched borrowers as \( e^b_F \) and \( e^v_t \), respectively. Each unmatched borrower seeks to procure \( P_t \) dollars incurs a cost of search effort given by \( a > 0 \).

Household loan agents with idle funds and borrowers seeking loans are brought together by a constant returns to scale matching technology that uses the number of unmatched agents on both sides of the market to produce a quantity of lending relationships: \( \tilde{G}_t = G(\tilde{v}_t, \tilde{L}_t) \), where \( \tilde{G} \) is strictly increasing and concave in each argument. This "black box" matching technology is meant to capture the informational frictions and firm-specific heterogeneity which make establishing lending relationships difficult. Specifying the ratio of unmatched borrowers to lenders as \( \theta_t = \tilde{v}_t/\tilde{L}_t \), we can express the probability of a borrower successfully matching with a
lender as \( \tilde{G}_t/\tilde{v}_t \equiv q^F(\theta_t) \) and the probability of a lender successfully matching with a borrower as \( \tilde{G}_t/\tilde{v}_t \equiv q^H(\theta_t) \), where \( \partial q^H/\partial \theta > 0 \) and \( \partial q^F/\partial \theta < 0 \). Hence \( \theta \) is measure of credit market tightness from the perspective of unmatched borrowers. The probability that an unmatched borrower (lender) finds a lender (borrower) is decreasing (increasing) in the aggregate number of unmatched borrowers and increasing (decreasing) in the aggregate number of unmatched lenders. These probabilities, which embody congestion externalities, are taken as given in the optimal decisions of households and firms. Also, as the flow of matched borrowers and lenders into the loanable funds market must be identical we have \( \tilde{G}_t = q^H(\theta_t)\tilde{l}_t = q^F(\theta_t)\tilde{v}_t \). Once a credit relationship is identified the terms of the loan contract which pays lenders a nominal interest rate \( R_t \) is determined by a bargaining solution which divides the match surplus between borrowers and lenders. At the beginning of each period there is an exogenous probability \( \psi \in (0, 1] \) that a credit relationship is dissolved. Hence, in real terms, the stock of matched credit relationships for lenders and borrowers evolves according to the following laws of motion:

\[
\tilde{l}_t^H = \left( \frac{P_{t-1}}{P_t} \right) (1 - \psi) \tilde{l}_{t-1}^H + q^H(\theta_t)\tilde{l}_t \\
\tilde{l}_t^F = \left( \frac{P_{t-1}}{P_t} \right) (1 - \psi) \tilde{l}_{t-1}^F + q^F(\theta_t)\tilde{v}_t.
\]

These say that the real total stock of credit relationships is the real quantity of unseparated relationships from the previous period and the flow of new matches. Notice that since each lending relationship corresponds to a value of \( P_t \) dollars, the net physical separation rate of credit relationships, \( \left( \frac{P_{t-1}}{P_t} \right) (1 - \psi) \), accounts for depreciation resulting from both exogenous separation and inflation which erodes the nominal value of funds carried over from the previous period. It will be analytically convenient to express these lending relationships in nominal terms. Let the nominal quantity of funds attached to existing lending relationships for lenders and borrowers be given by \( B_t^H = P_t\tilde{l}_t^H \) and \( B_t^F = P_t\tilde{l}_t^F \), the nominal quantity of unmatched loanable funds and borrower search effort be \( L_t = P_t\tilde{l}_t \) and \( V_t = P_t\tilde{v}_t \), and the nominal flow of loan creation be \( G_t = P_t\tilde{G}_t \). Hence we can express the nominal evolution of credit relationships as

\[
B_t^H = (1 - \psi)B_{t-1}^H + q^H(\theta_t)L_t, \quad (1) \\
B_t^F = (1 - \psi)B_{t-1}^F + q^F(\theta_t)V_t. \quad (2)
\]

where
\[ G_t = G(V_t, L_t), \]  
\[ q_t^H = G_t/L_t = q^H(\theta_t) \]  
\[ q_t^F = G_t/V_t = q^F(\theta_t) \]  

and \( \theta_t = V_t/L_t \).

We close with a description of the monetary authority. Denoting the nominal stock of money in period \( t \) as \( M_t \) the monetary authority injects currency \( X_t \) into the economy at a stochastic rate \( x_t \):

\[ M_{t+1} = M_t + X_t = (1 + x_t)M_t, \]  

This cash injection is received by the household’s loan agents. Hence the total number of unmatched loan agents \( \tilde{I}_t \) are funded from two sources: agents carrying the deposits of the household, \( \tilde{d}_t \), and those carrying the monetary injection \( \tilde{x}_t \). Hence, \( \tilde{I}_t = \tilde{d}_t + \tilde{x}_t \), or in nominal terms,

\[ L_t = D_t + X_t, \]  

where \( D_t = P_t \tilde{d}_t \).

\section*{2.2 Sequence of Events}
Household and firms are lumped together into an extended representative family which separate at the beginning of the period and reunite at the end to pool cash resources. At the beginning of the period the representative family begins with a nominal stock of money balances \( M_t \) and an existing stock of credit relationships attached to household and firms, \( B_{Ht}^1 \) and \( B_{Ft}^1 \), and a fraction of these \( \psi \) are dissolved. Households allocate total deposits \( TD_t \) among its loan agents. A portion of these deposits are pre-committed to loan agents who are funding existing matches and a portion \( D_t \) is allocated to unmatched lenders. Unmatched lenders in the financial market also receive loanable funds from the monetary injection \( X_t \) so that the total quantity of unmatched funds is \( L_t \) satisfies (7). We consider two alternative timing of events: having the allocation of deposits being made either before or after the realization of the monetary shock. As each unmatched loan agent has a probability \( q_t^H \) of locating a suitable borrower the aggregate amount of newly matched funds is \( q_t^H L_t \), the total stock of household credit is given by (1). The cost of search for households is thus the opportunity cost of idle funds held by unmatched loan agents at the end of the period, \( (1 - q^H)D_t \). Households
then proceed to the goods market and purchase consumption goods with their net-of-deposit money holdings $M_t - TD_t$ while workers supply labor and earn nominal wage $W_t$.

Firms allocate a nominal quantity of borrowers $V_t$, each incurring a real cost $a$ in the search for loanable funds. With the probability $q_t^F$ of locating a lender the aggregate amount of newly matched funds is $q_t^F V_t$ and the total stock of borrowed funds is given by (2). Newly established matched are immediately available to provide loanable funds. Firms hire labor $n_t^d$ on the competitive labor market, finance their wage bill with existing credit relations and the additional borrowed funds procured from the financial market, and produces output $Y_t$ at the end of the period interest payments $R_t$ are paid by borrowers to lenders, the family reunites and pool cash receipts.

2.3 Optimization and Equilibrium

The representative family chooses an optimal sequence of choice variables for households \{c_t, n_t, D_t\} and firms \{n_t^d, V_t\} which solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t)$$

subject to

$$P_t c_t \leq M_t - TD_t,$$  \hspace{1cm}  (8)

$$TD_t = (1 - \psi) B_{t-1}^H + D_t,$$  \hspace{1cm}  (9)

$$B_t^F \geq W_t n_t^d,$$  \hspace{1cm}  (10)

$$M_{t+1} = [M_t + W_t n_t + R_t B_t^H - P_t c_t] + X_t + [P_t Y_t - W_t n_t - R_t B_t^F - aV_t],$$  \hspace{1cm}  (11)

given \{P_t, W_t, \theta_t\}, where $B_t^H$ and $B_t^F$ are given by (1), (2), $L_t$ is given by (7), and $R_t$ is negotiated as the outcome of a bargaining solution described below. $E_0$ is the expectations operator conditional on information at time 0. Equations (8) and (9) represent the cash constraint for households, (10) is the financing constraint for firms, and the budget constraint (11) says that the nominal money balances of the family carried forward consists of the cash receipts of the household, the cash injection, and the cash receipts of firm profits. The market-clearing conditions for the goods, labor, credit, and money markets, respectively, are:
\[ Y_t = c_t + aV_t/P_t, \]  
\[ n_t = n_t^d, \]  
\[ B_t^H = B_t^F, \]  
\[ M_t = M_t^s = (1 + x)M_{t-1}^s. \]  

Notice that the model nests a special case of a frictionless credit market: when the matching probabilities \( q^H \) and \( q^F \) \( \to 1 \), firm search costs \( a = 0 \), and credit relationships depreciate after one period, \( \psi = 1 \), we have a standard limited participation framework.

In a stationary equilibrium nominal variables will grow at the same rate as the nominal money supply. Hence we scale nominal variables by the beginning-of-period money stock \( M_t \) and express these as lower case letters. That is, \( m = M/M^s, p = M/M^s, w = W/M^s, d = D/M^s, l = L/M^s, v = V/M^s, g = G/M^s, b^H = B^H/M^s, b^F = B^F/M^s, \theta = v/l, \) and \( x = X/M^s \). Also, we will suppressing time subscripts and denoting "-1" as last period variables and prime(’) as next period’s variables. With state space \( S \) the economy’s state vector is given by \( s = (m, b^H_{-1}, b^F_{-1}, x_{-1}, x) \in S \), and let the transition density be expressed as \( \Phi(s, ds') = \text{Prob}(s' = s_i \mid s = s_j) \) for all \( s_i, s_j \in S \). We can express the dynamic program for this problem as choosing \( \{c, n, d\} \) and \( \{n^d, v\} \) which solves

\[
J(m, b^H_{-1}, b^F_{-1}) = \max_d \int \max\{u(c, 1 - n) + \beta J(m', b^H, b^F)\} d\Phi(s, ds')
\]

subject to

\[
\begin{align*}
 pc & \leq m - [(\frac{1 - \psi}{1 + x_{-1}})b^H_{-1} + d], \\
 b^F & = \omega pn^d, \\
 m' & = (\frac{1}{1 + x})[m + \omega pn + Rb^H - pc + x + pY - \omega pn^d - Rb^F - av].
\end{align*}
\]

where \( \omega \) is the real wage, (16) is obtained after substituting (9) into (8) and scaling by \( M^s \), and (1) and (2) are given by

\[
\begin{align*}
 b^H & = (\frac{1 - \psi}{1 + x_{-1}})b^H_{-1} + q^H(\theta)(d + x) \\
 b^F & = (\frac{1 - \psi}{1 + x_{-1}})b^F_{-1} + q^F(\theta)v
\end{align*}
\]

Placing \( d \) outside the expectations operator reflects the limited participating version where it is chosen before the state \( s' \) is revealed.
Letting $\lambda_1$ and $\lambda_2$ denote the Lagrange multipliers on (16) and (17), respectively, the first order conditions for $\{c, n, d, n^d, v\}$ are given by

\[
\frac{u_c(c, 1-n)}{p} = \beta J_m(s') + \lambda_1
\]  
(21)

\[
\frac{u_L(c, 1-n)}{p} = \beta J_m(s') \omega
\]  
(22)

\[
\int \left\{ \frac{\beta J_m(s')}{1+x} R q^H + \beta J_{bh}(s') q^H \right\} d\Phi = \int \lambda_1 d\Phi
\]  
(23)

\[
\beta \frac{J_m(s')}{1+x} [R q^F + a] = \lambda_2 q^F + \beta J_{bF} q^F
\]  
(24)

and the envelope (co-state) conditions are

\[
J_m(s) = \int \left\{ \frac{\beta J_m(s')}{1+x} + \lambda_1 \right\} d\Phi
\]  
(26)

\[
J_{bh}(s) = \int \Psi \left\{ \frac{\beta J_m(s')}{1+x} R - \lambda_1 + \beta J_{bh}(s') \right\} d\Phi
\]  
(27)

\[
J_{bF}(s) = \int \Psi \left\{ \lambda_2 - \beta \frac{J_m(s')}{1+x} R + \beta J_{bF}(s') \right\} d\Phi
\]  
(28)

where $u_c = \partial u/\partial c$, $u_L = \partial u/\partial (1-n)$, $J_m(s) = \partial J(s)/\partial m$, $J_{bh}(s) = \partial J(s)/\partial b^H$, $J_{bF}(s) = \partial J(s)/\partial b^F$, and $\Psi(s) \equiv (1 - \psi)/(1 + x_{-1})$.

### 2.3.1 Efficiency Conditions

Equations (21) and (26) imply that the marginal value of an extra dollar carried into the current period is simply the expected marginal value of consumption:

\[
J_m(s) = \int \frac{u_c}{p} d\Phi
\]  
(29)

Substituting this into (22) gives that the efficiency condition for work effort which equates the utility lost from an extra unit of work effort with the discounted future marginal benefit of consumption financed by extra wage income:

\[
u_L = p\omega \left( \frac{\beta}{1+x} \right) \int \frac{u_c}{p} d\Phi
\]  
(E1)

Equation (21) gives the value of $\lambda_1$, which measures the extent to which cash constraint
(16) binds, as the difference in the marginal value of current to discounted value of future consumption:

$$\lambda_1 = \frac{u_c}{p} - \beta \frac{J_m(s')}{1 + x}$$

(30)

Combining this into (23) says that along the optimal path, household deposits must satisfy:

$$\int \left\{ \beta \frac{J_m(s')}{1 + x} [1 + R q^H] + \beta J_{\nu u}(s') q^H \right\} = \int \frac{u_c}{p} d\Phi$$

(31)

Again, a special case of this condition when $$q^H = 1 = \psi$$, which implies $$J_{\nu u}(s') = 0$$, is the Fisherian decomposition of the nominal interest rate into a real component, given by expected consumption growth, and an expected inflation component. Equations (23) and (27) can be combined as

$$J_{\nu u} (s) = \int \Psi \lambda_1 \left( \frac{1 - \frac{q^H}{q^H}}{q^H} \right) d\Phi$$

(32)

which says that along the optimal path the marginal value of an existing a credit relationship for the lender is the expected marginal benefit the undepreciated portion provides in avoiding the cost of searching for new lending opportunities. This cost is increasing in both $$\lambda_1/q^H$$, the extent to which the cash constraint (16) binds when $$(1/q^H)$$ dollars of deposits create an extra dollar of loans, and in $$(1 - q^H)$$, the end-of-period opportunity cost of unmatched funds. Substituting (32) back into (31) and using $$\lambda_1$$ from (21) gives us the household efficiency condition for deposits ($$d$$):

$$\int \left\{ \beta \frac{J_m(s')}{1 + x} [1 + R q^H] + \beta q^H \Psi \left[ \frac{u_c}{p} - \beta \frac{J_m(s'')}{1 + x'} \right] \left( \frac{1 - \frac{(q^H)'}{(q^H)''}}{(q^H)''} \right) \right\} d\Phi = \int \frac{u_c}{p} d\Phi.$$  

(E2)

The left hand side of this expression is the expected marginal benefit of an extra dollar of deposits, which is the sum of the interest income earned from having a fraction $$q^H$$ successfully matched with borrowers and the discounted future value of the credit relationship described above, and the right hand side is the marginal value of current consumption sacrificed.

Solving for $$\lambda_2$$ in (25) shows that the marginal cost of an extra dollar procured in the loan market is equated with the benefit of hiring $$(1/\omega)$$ units of labor which supplements the family’s cash receipts with additional profits:

$$\lambda_2 = \beta \frac{J_m(s')}{1 + x} \left[ \frac{f_n(n) - \omega}{\omega} \right]$$

(33)

Substituting this into (28) gives

$$J_{\nu F} (s) = \int \Psi \beta \frac{J_m(s')}{1 + x} \left( \frac{a}{q^F} \right) d\Phi,$$

(34)
which says the marginal value of an existing credit relationship for the borrower is the expected marginal benefit the undepreciated portion of those funds provides in avoiding cost of searching for new matches. Since an extra dollar of borrowed funds from a successful match requires firms to allocate \((1/q^F)\) unmatched borrowers, the marginal cost is \((a/q^F)\). Combining (34) with (28) gives us the efficiency for borrower search effort \((v)\):

\[
\beta \frac{J_m(s')}{1 + x} \left[ R + \frac{a}{q^F} \right] = \beta \frac{J_m(s')}{1 + x} \left[ \frac{f_n(n) - \omega}{\omega} \right] + \beta \int \psi' \beta J_m(s'') \frac{a}{1 + x (q^F)} d\Phi
\]

(E3)

The left hand side of (E3) is the marginal cost of a successful match that procures an extra dollar of loanable funds, which requires \((1/q^F)\) units of search effort costing \(a\) and the interest cost \(R\) associated with the credit relationship, with the marginal benefits of hiring \((1/\omega)\) units of labor and the continuation benefit of the match.

### 2.3.2 Interest Rate Determination

To close the characterization of our equilibrium conditions this section describes the determination of interest rates corresponding to loan contracts in the financial market. A natural mechanism employed in determining the terms of trade in relationships established in a matching market is the Nash Bargaining solution which splits the surplus value of the match between borrowers and lenders.\(^5\) To implement the solution we first need to determine the value of a match in the loanable funds market for household/lenders and firm/borrowers. The match surplus for lenders in the current period is given by

\[
S_H = \partial J(s)/\partial b^H = \beta \frac{J_m(s')}{1 + x} R - \lambda_1 + \beta J_{bH}(s') = \lambda_1 \left( \frac{1 - q^H}{q^H} \right) = \left[ \frac{u_c}{p} - \beta \frac{J_m(s')}{1 + x} \right] \left( \frac{1 - q^H}{q^H} \right)
\]

(35)

That is, the value of an extra dollar matched in the credit market for the lender is the expected discounted value of flow of interest income and the continuation value of the match less the opportunity cost of funds carried by unmatched loan agents at the end of the period. Using (32) and (30) shows this value must be equated with the expected future search cost of idle funds avoided by establishing a current match.

Similarly, the match surplus for borrowers is given by

\[
S_F = \partial J(s)/\partial b^F = \lambda_3 - \beta \frac{J_m(s')}{1 + x} R + \beta J_{bF}(s') = \beta \frac{J_m(s')}{1 + x} \left( \frac{a}{q^F} \right)
\]

(36)

\(^5\)Bargaining solutions have been used extensively in matching models of the labor market (e.g., Mortensen and Pissarides (1994)) as well as the recent literature on credit market search (e.g., Wasmer and Weil (2004), Becsi, Li, and Wang (2005, 2008).
The value of an extra dollar matched in the credit market for the borrower is the expected discounted value of profits from production (see (25)) and the continuation value of the match less the flow interest costs of the borrowed funds. Optimal search effort implies that this value must be equated with the expected future search cost avoided by establishing a current match.

The total match surplus is given by $TS = S_H + S_F = (\lambda_3 - \lambda_1) + \beta[J_{bH}(s') + J_{bF}(s')]$. The first term in brackets is the current returns to a successful match. From (30) and (33) this is equal to $[\beta J_m(s')/(1+x)][f_n(n)/\omega] - u_c/p$ or the difference between the expected future benefits of production made possible by loanable funds and the cost of sacrificed current consumption. The second term in brackets is the total expected search cost avoided by borrowers and lenders from establishing a long-term lending relationship. The division of the match surplus between borrowers and lenders is determined by choosing $R$ to maximize $S_H^{(1-\gamma)}$, where $\gamma \in [0,1]$ represents the bargaining share of lenders and this yields the solution $(1 - \gamma)S_H = \gamma S_F$. This outcome implies that factors which make the match surplus more valuable to lenders $S_H$, such as a greater opportunity cost of unmatched funds ($\lambda_1$), a higher continuation value of the match ($J_{bH}$) or a lower probability of finding borrowers ($q^H$) will decrease $R$. Similarly, factors which increase the match value for borrowers, such as a higher continuation value $J_{bF}$ or lower probability of finding lenders ($q^F$) will increase $R$. Substituting (35) and (36) into the bargaining solution yields

$$\frac{u_c}{p}(1 - q^H) = \beta \frac{J_m(s')}{1 + x} [(1 - q^H) + \tilde{\gamma} \alpha \theta].$$

(E4)

where $\tilde{\gamma} \equiv \gamma/(1 - \gamma).

2.3.3 Stochastic Equilibrium

Since money and credit market equilibrium implies $m = m' = 1$, and $b^H = b^F = b = \Psi b_{-1} + g(v, d + x)$, (16) and (12) can be combined to solve for $c$ and $p$ as a functions of $n, v, d, b_{-1}$:

$$c = \frac{f(n)[1 - \Psi b_{-1} - d]}{av + [1 - \Psi b_{-1} - d]}$$

(37)

$$p = \frac{av + [1 - \Psi b_{-1} - d]}{f(n)}.$$  

(38)

Substituting these into (17) gives us the equilibrium real wage rate as

$$\omega = \frac{f(n)[\Psi b_{-1} + g(v, d + x)]}{[av + (1 - \Psi b_{-1} - d)]n}$$

(39)
From (22) we can express the discounted value of cash carried into next period as $\beta J_m(s')/(1 + x) = u_c/p\omega$ and substitute it into our efficiency conditions. Given state variables $\{b, x, x\}$ we can define an equilibrium as $\{n, d, v, R\}$ satisfying

$$\frac{\beta}{1 + x} \int \frac{u'_c}{p'} d\Phi = \frac{u_c}{p\omega}$$

$$\int \left\{ \frac{u_c}{p\omega} \left[ 1 + Rq^H \right] + \beta q^H \Psi' \left[ \frac{u'_c}{p'} - \frac{u'_c}{p'\omega'} \right] \left( \frac{1 - (q^H)'}{(q^H)'} \right) \right\} d\Phi = \int \frac{u_c}{p} d\Phi$$

$$\frac{u_c}{p\omega} \left[ R + \frac{a}{q^F} - \left( \frac{f'(n)}{\omega} \right) \right] = \beta \int \left\{ \Psi' \frac{u'_c}{p'\omega'} \left( \frac{a}{(q^F)'} \right) \right\} d\Phi$$

$$u_c(1 - q^H) = \frac{u_c}{\omega} \left[ (1 - q^H) + \tilde{\gamma} a\theta \right].$$

Where $c, p, \text{and } \omega$ are given by (37), (38), and (39), $q^H = g/(d+x)$, $q^F = g/v$, and $\theta = v/(d+x)$.

2.3.4 Steady State

To obtain a more concrete understanding of the model's steady state, as well facilitating the quantitative exercises below, we will adopt some specific functional forms for preferences and technology: $Y = f(n) = zn^x$, where $z > 0$ and $0 < \alpha < 1$, and $u(c, 1-n) = u_0 \ln(c) + A(1-n)$, where $u_0 > 0$ and $A > 0$. Given our functional forms the steady state versions of (40), (41), (42), and (43) can be expressed as $\{n^*, d^*, v^*, R^*\}$ solving

$$\frac{\beta}{1 + x} \frac{u_0}{c} = \frac{A}{\omega}$$

$$\frac{\beta}{1 + x} [1 + Rq^H - \beta \Psi (1 - q^H)] = [1 - \beta \Psi (1 - q^H)]$$

$$\frac{a}{q^F} (1 - \beta \Psi) = \left[ \frac{\alpha n^{x-1}}{\omega} - (1 + R) \right]$$

$$\frac{u_0(1 - q^H)}{c} = \frac{A}{\omega} [(1 - q^H) + \tilde{\gamma} a\theta].$$

Where $c, p, \text{and } \omega$ satisfy (37), (38), and (39) and $b = g(v, d + x)/(1 - \Psi)$. With these we make the following observations regarding the steady state:

- The steady interest rate $R^*$ is decreasing in $q^H$ and for any $q^H < 1$, it is greater than the Fisherian fundamental rate of $\hat{R} \equiv (1 + x)/\beta - 1$.

- The steady state values for $\{d^*, v^*\}$ are determined independently of $n^*$. Given unique $\{d^*, v^*\}$, there exists a unique value for $n^*$. 

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The first observation comes directly from (S2). Simplifying the expression yields the nominal interest rate as a function of $q^H$:

$$R^* = \left[\frac{1 - \beta \Psi(1 - q^H)}{q^H}\right] \left(\frac{1 + x - \beta}{\beta}\right) > \left(\frac{1 + x - \beta}{\beta}\right) = \hat{R}. \quad (44)$$

This implies that, all else equal, the optimal deposits decision of lenders leads to an inverse relationship between the probability of a successful match with a borrower and the nominal rate. One could interpret this as a "risk" premium to compensate lenders with the possibility that funds will leave the financial market unmatched. Only in the case of an absence of search frictions will $q^H = 1$ and $R^*$ will be driven entirely by Fisherian Fundamentals.

The second observation comes from substituting (44) and (39) into (S3) which gives

$$\frac{a}{q^H}(1 - \beta \Psi) = \frac{\alpha[av + 1 - \Psi b - d]}{b} - 1 - Z \left(\frac{1 + x - \beta}{\beta}\right). \quad (45)$$

where $Z = [1 - \beta \Psi(1 - q^H)]/q^H$. Notice that (S4) and (45) solve $\{d^*, v^*\}$ independently of the value of $n$. Since the steady state bargaining solution (S4) is a strictly upward sloping locus $(d^*, v^*)$, a sufficient condition for the uniqueness of this steady state is an inverse relationship between $v$ and $d$ implied by (45). Given these values, substituting (37) and (39) into (S1) yields a unique value for $n^*$:

$$n^* = \frac{b}{A} \left(\frac{\beta}{1 + x} \frac{u_0}{1 - \Psi b - d}\right). \quad (46)$$

Notice that all else equal, greater participation in the financial market (higher $v$ and $d$) increases labor market participation ($n^*$). Additional properties of the steady state will be studied in the quantitative analysis of the next section.

### 3 Quantitative Exercises

This section will characterize some properties of the steady state and stochastic equilibrium and numerically simulate the model to access some quantitative implications of financial market search relative to a standard frictionless framework. In addition to the functional forms for preferences and production technology used in the steady state analysis above, we follow den Haan, Ramey, and Watson (2000) and Nicoletti and Pierrard (2006) and adopt a constant returns to scale matching technology given by

$$g(v, l) = z^g \frac{vl}{[v^h + l^h]^{1/h}}. \quad (47)$$
where $h > 0$ and $z^g > 0$ is a productivity parameter capturing the efficiency of the matching technology. The corresponding matching probabilities can be derived by first defining

$$h(\theta) = \frac{z^g}{[1 + \theta^h]^{1/h}}.$$ 

Therefore, $q^F(\theta) = g/v = h(\theta)$ and $q^H(\theta) = g/l = \theta h(\theta)$, $\partial q^H/\partial \theta > 0$ and $\partial q^F/\partial \theta < 0$. This specification offers two convenient features. First, $h$ captures the degree of search frictions in the financial market. Taking $h \to 0$ implies severe frictions while $h \to \infty$ gives $g \to \min\{v, l\}$ and the absence of frictions. Secondly, the matching probabilities are always bounded between 0 and 1 which makes the numerical analysis more convenient. Otherwise, the results are not sensitive to this particular specification relative to the standard Cobb-Douglas technology.

While the lack of a capital market simplifies our analysis and isolates the effects of financial market search it also precludes a more comprehensive quantitative study. Our focus here is not to conduct a serious calibration exercise to match moments of the data, but rather to better understand some quantitative properties of the model and the contribution of costly credit market search. Hence some of our benchmark model parameters will be chosen to match observed steady state outcomes while others will be used for sensitivity analysis. Consistent with conventional business cycle studies we set $\alpha = 0.64$ and $\beta = 0.99$. The disutility of labor $A$ is set to give a steady state work effort of $n$ to be $1/3$. Also, as we noted above steady state $v$ and $d$ are invariant to the choice of $A$. Following Hendry and Moran (2004) and Nicoletti and Pierrard (2006) we set the matching function parameter so that $q^F = q^H = 0.75$, so $\theta = 1$. In this model these values are $h$ to 2.41 and firm search cost $a$ to 0.00555. In a quarterly model this implies that the average time for borrowers to find lenders and vice versa to be about one quarter. The bargaining share of lenders, $\gamma$ is set to 0.5, which gives a steady state nominal interest rate of 2.3%, and following Christiano (1991) and Dressler and Li (2009) the steady state money growth rate is set to $x^r = 0.012$. We set productivity parameters $z$ and $z^g$ to 1.

### 3.1 Anticipated Inflation & the Steady State

Using the parameterization described above we first study the impact of steady money growth and inflation for credit market activity and aggregate real variables. Figure 1 shows the effect of increasing the money growth rate $x$ from our benchmark value of 1.2% to 5.7% on steady state variables. With the exception of the nominal interest rate and credit market tightness the axis measures the variable as ratios to the benchmark value. Not too surprisingly, higher money growth rates lead to higher nominal interest rates. For a given value of $q^H$ this arises from equation (44) which embodies the traditional anticipated inflation effect on nominal rates.
However, the decline in household deposits \((d)\) and the reduction of real loanable funds \((l/p)\) increases credit market tightness \((\theta)\) making it more difficult for borrowers to locate lenders. While this tends to amplify the higher rates by shifting bargaining power towards lenders, the increase \(q^H\) tends to mitigate the traditional inflation tax effects in (44).

An interesting feature is that borrowers incur the real costs of additional search effort \((v/p)\) in response to the increased difficulty of finding lenders and the real erosion of existing credit relationships. One can think of this "loan market participation" effect as a type of "shoe leather" cost of high inflation to borrowers. Consequently, the inflow of credit creation \((g/p)\) increases. The real stock of lending \((b/p)\) in the financial market declines and, consistent with conventional monetary models, we find that aggregate real variables, labor, consumption, and output, also decreases with higher steady inflation. This positive effect of inflation on the size of the banking and credit sector and loan market participation is consistent with evidence documented by Aiyagari, Braun, and Eckstein (1998), English (1999), and the negative impact of inflation on the real quantity of lending activities by Boyd and Champ (2006). The case of \(\psi = 1\) shows a similar pattern with comparable increases in the nominal interest rate and decline in real loanable funds and aggregate variables. A notable difference is that while firm search effort continues to increase with the inflation rate the inflow of new credit relationships declines. Also, long-term credit relationships (for the case of \(\psi = 0.10\)) tends to strengthen the degree by which steady inflation increases the nominal rate and decreases real deposits. We will discuss this finding further below.

### 3.2 Monetary Shocks

This section investigates the quantitative implications of our general model to stochastic innovations in the money growth rate. We will consider both the case with and without the limited participation constraint and compare our findings with the standard model in the absence of financial market search frictions. The money growth rate is assumed to follow a stationary AR(1) process:

\[
x_{t+1} = (1 - \rho)x^* + \rho x_t + \varepsilon_{t+1}
\]

where \(x^*\) is the steady state money growth rate, \(0 < \rho < 1\) is the degree of persistence of the monetary shock, and \(\varepsilon_t\) is an white noise disturbance with zero mean and constant variance. Again, consistent with previous studies, we benchmark \(x^* = 0.012\) and \(\rho = 0.32\). To isolate pure liquidity effects from anticipated inflation we will also consider i.i.d. shocks to the money growth rate \((\rho = 0)\).
Reverting back to time subscript notation, expressing the marginal value of cash as the expected future marginal value of consumption given by (29), and denoting $E_t$ as the time-$t$ expectations operator, notice that our efficiency conditions (E1)-(E4) can be expressed as

$$E_{t-1}\left\{ \frac{\beta}{1+x_t} E_t \left\{ \frac{u_{c,t+1}}{p_{t+1}} \right\} = \frac{A}{p_t \omega_t} \right\}$$

$$E_{t-1}\left\{ \frac{\beta}{1+x_t} E_t \left( \frac{u_{c,t+1}}{p_{t+1}} \right) \left[ 1 + R \frac{q^H}{\Psi_{t+1}} \right] \left[ \frac{u_{c,t+2}}{p_{t+2}} \right] = E_{t-1} \left\{ \frac{u_{c,t+1}}{p_{t+1}} \right\} \right\}$$

$$1 + R \frac{a}{p^F} \left[ 1 + x_t \right] E_t \left\{ \frac{u_{c,t+1}}{p_{t+1}} \right\} = \left[ f_t(n_t) \right] \frac{1}{\omega_t} E_t \left( \frac{u_{c,t+1}}{p_{t+1}} \right)$$

$$E_t \left\{ \Psi_{t+1} \left[ \frac{u_{c,t+1}}{p_{t+1}} \right] \left[ \frac{u_{c,t+2}}{p_{t+2}} \right] = \frac{a}{p^F} \right\}$$

$$u_{c,t} \left( 1 - q^H_t \right) = \frac{A}{\omega_t} \left[ (1 - q^H_t) + \gamma a \theta_t \right]$$

where $\Psi_t \equiv (1 - \psi)/(1 + x_{t-1})$, $c_t$, $p_t$, and $\omega_t$ are given by

$$c_t = \frac{f_t(n_t)[1 - \Psi b_{t-1} - d_t]}{a v_t + [1 - \Psi b_{t-1} - d_t]}$$

$$p_t = \frac{[1 - \Psi b_{t-1} - d_t]}{c_t}$$

$$\omega_t = \frac{b_t}{p_t m_t}$$

Notice that the $E_{t-1}$ expectations operator in equation (50) captures the limited participation timing of deposit decisions. The equilibrium conditions are linearized about the benchmark steady state to yield decision rules linear in the state vector $\{b_{t-1}, x_{t-1}, x_t\}$. We consider two versions of our credit model: non-limited participation (NLP) where $d_t$ is chosen after the monetary shock and limited-participation (LP) where it is chosen before. We also compare our results relative to the model in the absence of financial market search frictions (i.e., the baseline monetary model), where $q^H = q^F = 1$, $\psi = 1$ and $a = 0$, with and without the limited participation constraint (we’ll refer to these as BLP and BNLP). The financial market of our model differs from the frictionless case in two respects. First, there is costly search and matching and second there are long-term credit relationships. To study the contribution of each to our results we consider a benchmark separation rate at $\psi = 0.10$ and also where, while search is still costly, there are no long-term relations ($\psi = 1$). The impulse response plots contained in Figures 1 - 7 show the dynamic impact of a 1% shock to the money growth rate in period 5. With the exception of the nominal interest rate (which shows deviation from
3.2.1 Non-Limited Participation Case

We first examine the model when \( d_t \) is chosen after the monetary shock is realized. Consider the case of an i.i.d positive monetary innovation \( (\rho = 0) \). For both our credit NLP model with \( \psi = 1 \) and the frictionless BNLP these money shocks are neutral. Cash is allocated optimally so that its marginal value is equalized across the goods and financial market and, as there are no anticipated inflation effects, nominal deposits in both models decrease in the period of the shock as the extra cash is funneled out of the financial market. Hence, costly search in the financial market leaves this basic result unaltered. However, in the credit NLP model any value of \( \psi < 1 \) and the existence of long-term credit relationships renders the money shock non-neutral. Figure 2A and B illustrates this for \( \psi = 10\% \). From (E2) this is because the marginal value of deposits is greater in the financial market when credit long term credit relationships are established \( (\Psi > 0) \) and a greater withdrawal of deposits are necessary to equate it with the marginal value of cash in the goods market relative to BNLP. The decline in deposits leads to a tighter credit market (increase in \( \theta \)) and a higher match value to borrowers, which implies that the nominal interest rate \( R \) rises in the period of the shock. The expected decline in nominal rates in the following period lowers the cost of financing long-term credit relationships and increases borrower search effort \( (v/p) \) and overall investment in credit \( (g/p) \). The decline in the stock of real credit relationships \( (b/p) \) is only rebuilt over time. Consequently, there is a decrease labor, real consumption and output which persists in the periods following the shock. Therefore, the response of NLP is similar to an anticipated inflation effect even though the money shock exhibits no persistence. Let’s call this the "long-term lending" effect.

Now consider the case where there is persistence in the money growth process \( (\rho = 0.32) \). Figure 3A and B illustrates the responses of both BNLP and NLP for the case of \( \psi = 1 \). The anticipated inflation effect on these variables looks very similar to our results in the steady state. That is, the nominal rate increases, there is a decline in household deposits \( (d) \), an increase in borrower search effort \( (v/p) \), a tighter credit market \( (\theta) \), decrease in credit creation \( (g/p) \) and a decline in the real stock of loans \( (b/p) \). Notice that the credit market remains tight for several periods after the shock and converges back to steady state from above. There is an increase in \( R \) and a decrease in real activity. Notice, however, this effect is much smaller than NLP and the reason is as follows. There is no long-term lending effect described in the i.i.d. case above. Anticipated inflation shocks, however, increase the value of a match for lenders seeking borrowers by raising the opportunity cost of having unmatched idle funds at the end of the period. This "bargaining effect" works against the traditional inflation tax as it
places downward pressure on interest rates. Hence costly credit market search and bargaining
without long-term credit relations mitigates the conventional anticipated inflation effect.

Finally, we introduce long-term credit relations by setting $\psi = 0.10$ and combine this with
a persistent money growth shock ($\rho = 0.32$). Figure 4A and B plots the results of NLP against
case above where $\psi = 1$. Notice now that monetary shocks are associated with traditional
anticipated inflation effects as well as the bargaining and long-term lending effect. Hence, the
size of the anticipated inflation effect is larger relative to $\psi = 0$. Consistent with our steady
state findings we see a larger increase in firm search effort, a greater flow of credit creation
while a lower the stock, and increased credit market tightness.

3.2.2 Limited Participation

To analyze the impact of financial market search on liquidity effects we consider the alternative
timing assumption where $d_t$ is chosen before the monetary shock is realized (the LP model).
By creating a differential between the marginal value of cash in the goods and financial market
which cannot be corrected within the period unexpected money injections will be associated
with the observed downward movement of the nominal interest rate.

First consider the case with $\rho = 0.32$ but no long-term credit relations ($\psi = 1$). As shown
in Figure 5 a positive money growth rate shock drives the nominal interest rate $R$ down in the
credit LP model. While nominal deposits are fixed in the period of the shock, financial market
participation in the form of additional real deposits ($d/p$) and borrower search effort ($v/p$)
increase which induces credit creation ($g/p$). Consequently, the higher quantity of loanable
funds finances additional labor and leads to an expansion of consumption and output.

One curious implication of the monetary shock is the increase in credit market tightness ($\theta$).
To see how this is consistent with our equilibrium conditions we can view the liquidity effect
within the period in three stages. First, all else being equal, the additional funds injected
into the market should decrease credit market tightness and, from the bargaining solution,
place downward pressure on the interest rate. Notice that when $\psi = 1$ the optimal condition
for borrower search effort (41) says that the current marginal cost of search must be equated
with the benefit of financing current production: $[1 + R] = f_n/\omega - a/q^F$. Hence a lower $R$
should correspond to an increase in $v$ by an amount greater than the new injection of loanable
funds, $d + x$, so that $q^F$ decreases and this leads to a tighter credit market. Finally, as the
additional loanable funds supports the expansion of labor and the real wage rate, there is some
mitigation of this effect through a decrease in the marginal product of labor. Hence a tighter
credit market will co-exist with greater credit creation, an increase in lending relationships,
and higher aggregate activity. Relative to the frictionless BLP model, also plotted in Figure 4,
the size of the liquidity effect is substantially larger. This arises directly from the bargaining
effect on borrowers (through the initial injection of liquidity) and lenders (through anticipated inflation), both of which suppress the traditional inflation tax effect.

Figure 6 plots the LP model for both the previous case where $\psi = 1$ and our benchmark $\psi = 0.10$. The presence of long-term credit relations now re-introduces the long term lending effect which dampens the size of the liquidity effect. The movement in deposits after the period of the shock is now much larger. The other qualitative effects are similar but notice now that several credit market variables, including firm search effort, credit creation, and credit market tightness all converge back from below their steady state values.

One problematic prediction of our benchmark (LP) model and the standard (BLP) model is the lack of any substantial persistence in liquidity effects on the nominal interest rate and real activity. This counterfactual result arises because the differential between the value of cash in goods and financial markets can be undone in the following period by withdrawing deposits from the financial market. Hence the nominal rate and real variables revert to the steady state almost immediately. One solution to this issue, first explored by Christiano and Eichenbaum (1992), is to add exogenous adjustment costs to deposit decisions which will make them sluggish in the period after the shock. Rather than impose these costs ex-ante we would like to investigate whether a similar mechanism can arise endogenously from our credit search model.

The matching technology assumes that tightness in the credit market is identical on both sides of the market ($\theta = 1$). While this was convenient in terms of interpreting results it may preclude other interesting model dynamics which link liquidity effects with the search and bargaining process. To this end, we consider an environment where credit market tightness is relative low by increasing the firm search cost value $a$ to 0.05. This gives $\theta = 0.31$ and the steady state value of $v$ is 70% of our benchmark model. A higher search cost also increases lender bargaining power and the steady state interest rate $R$ increases to 2.83%. As before we set $\psi = 0.10$ and $\rho = 0.32$. The impulse response plot for this exercise is given by Figure 7 where the "high search cost" case, HSC, is compared with our benchmark "low search cost" or LSC. While the liquidity effect continues to operate in response to the money shock an immediate difference is that the nominal interest rate falls for two periods and stays below its steady state value for several more. Labor, consumption, output, and the stock of credit display humped-shaped responses and rise for two periods before reverting back to their steady state values. Our credit market variables: loan creation, borrower search effort, and real loanable funds also stay above the steady state before dropping below and converging. Hence, the model is able to capture a persistent effect of a monetary shock.

To identify the source of this persistence notice that this is occurring in a highly liquid credit market with a high cost of search effort and very few borrowers searching for loanable
funds. The high cost of search implies that the steady state bargaining power for borrowers (lender) is low (high). As seen in other versions of our LP model considered above, credit market tightness increases in the period of the shock as the additional liquidity encourages borrower entry (search effort), and there will be a decrease in the value of $\theta$, an increase in $q^F$ and decrease in $q^H$ in the period following the shock. Equation (36) indicates that the negative marginal impact of the increase in $q^F$ on the borrower’s match surplus is increasing in the cost of search $a$ and hence bargaining implies there will be additional downward pressure on the nominal rate in the period after the shock. On the lender’s side, the damped decline in $q^H$ in the period after the shock leads to a high marginal value of deposits in the financial market and deposits will be optimally sluggish as the nominal rate declines further before returning to its steady state value. Ultimately, the transfer of bargaining power from lenders to borrowers is amplified in the period after the shock and creates a persistent liquidity effect. By the same logic, it is also verified that a similar persistent pattern can also be generated in the situation where the bargaining share of borrowers $(1-\gamma)$ is sufficiently low.

4 Agency Costs and Endogenous Separation

In our basic framework of financial market matching we assumed that the rate by which borrowers and lenders are separated at the beginning of each period, $\psi$, was exogenous. Such an assumption provided both simplicity and facilitated an analysis of the implications of long-term credit relationships to monetary policy relative to a baseline model with the absence of credit market frictions. However, it may also omit an important channel of monetary transmission which arises from informational based credit frictions that lead to the moral hazard problem emphasized in the seminal papers of Bernanke and Gertler (1989) and Carlstrom and Fuerst (1995). That is, the situation where financial distress coupled with asymmetric information may lead to an endogenous break-up of credit relationships. Incorporating these features into the model would not only enable us to address the documented countercyclical behavior of loan destruction over the business cycle, e.g. Craig and Haubrich (2006), but also study the impact of monetary policy on the equilibrium separation rate as a potential source of propagation.

Following Carlstrom and Fuerst (1995) assume that in addition to the aggregate level of productivity $z_t$, each firm/borrower faces an idiosyncratic productivity shock in period $t$, denoted by $e_t \in [0, \infty)$ which is private information and cannot be costlessly observed by the household/lender: $Y_t = e_t f(n_t)$. These shocks are i.i.d and drawn from a normal distribution with unconditional mean of 1 and constant variance. Realization of the shock occurs after optimal decision rules regarding consumption, labor demand and supply, nominal
deposits, firm search effort, and the interest rate bargain are made. The asymmetric information naturally leads to a moral hazard problem as borrowers have an incentive to report to lenders a productivity level below its true realized value and default. However, it is assumed that households have access to a monitoring technology which can verify the outcome of the firm’s project at a cost equal to a fraction \( \eta \) of the firm’s output. Hence, the loan contract which induces truthful revelation by firms is one were the household monitors the project if the idiosyncratic productivity shock is reported below an optimally chosen threshold value \( \bar{\epsilon}_t \) which satisfies

\[
P_t \bar{\epsilon}_t f(n_t) - W_t n_t - R_t B_t^F - aV_t = 0
\]

That is, if \( \epsilon_t > \bar{\epsilon}_t \), then the firm will earn positive profits and repay \( R_t B_t^F \) according to the loan agreement. However, if \( \epsilon_t < \bar{\epsilon}_t \) then the household will monitor to verify the outcome and the firm defaults. In the case of default, the firm is liquidated, the household pays both search and monitoring costs from the firm’s output, claims any residual, and desolves the match. Let \( \psi_1 \) be the exogenous separation rate. Denoting \( H(\epsilon) \) as the cumulative density of \( \epsilon \), the endogenous component of the separation rate can be expressed as

\[
\psi_{2t} = H(\bar{\epsilon}_t)
\]

and the total rate of separation is given by \( \psi_t = \psi_1 + \psi_{2t} \). Workers are paid from the proceeds of the loan and face no uncertainly regarding their labor income, so the cash constraint (10) remains unchanged. Incorporating the expected value of the household’s gross return to deposits, \( \tilde{R}_t \), and the firm’s expected profits in the case of repayment into the budget constraint, (11) becomes

\[
M_{t+1} = [M_t + W_t n_t + \tilde{R}_t - P_t c_t] + X_t + \left[ \int_{\bar{\epsilon}_t}^{\infty} \{ P_t \epsilon_t f(n_t) - W_t n_t - R_t B_t^F - aV_t \} dH(\epsilon_t) \right]
\]

where

\[
\tilde{R}_t = \int_{\bar{\epsilon}_t}^{\infty} R_t B_t^H dH(\epsilon_t) + \int_{0}^{\bar{\epsilon}_t} \{ (1 - \eta) P_t \epsilon_t f(n_t) - W_t n_t - aV_t \} dH(\epsilon_t)
\]

Equation (56) is the expected return to households. The first term is the expected return when the borrower repays the loan and the second term is the expected return in the event the borrower defaults and includes the cost of monitoring. The model is closed again by the
Nash bargaining solution over interest rate $R_t$ and market-clearing in the labor, financial, and goods markets.

A couple of conjectures are made about this agency cost extension of the model. First, from (44) and (46) the inclusion of agency costs which increases the separation rate (decreases $\Psi$) also increases the steady state interest rate, as lenders now demand an additional risk premium with the likelihood of default, and decreases steady state employment. Second, equations (53) and (54) will imply that monetary injections which create a liquidity effect and lowers nominal rates will also reduce credit separation rates in addition to increase matching rates. This will not only amplify the size of the liquidity effect but potentially create an additional propagation mechanism. Such a finding will also be consistent with the documented evidence supporting procyclical loan creation and countercyclical loan destruction rates. (To be completed)

5 Concluding Remarks

This paper integrates a decentralized financial market characterized by costly search and long-term credit relationships with a monetary business cycle model with limited participation. The model replicates the observation that steady inflation can increase loan market participation while disrupting the total stock of lending activity. Also, the interaction between loan market participation decisions, credit market tightness, bargaining, and long-term relationships alters the size of both the liquidity and anticipated inflation effects associated with monetary policy shocks relative to a frictionless model. Furthermore, these interactions also embody an internal mechanism capable of delivering persistence of liquidity effects which can last several periods after the monetary injection.

Our framework represents just one step in a broader research agenda seeking to evaluate the implications of financial market search frictions for monetary policy. The environment was purposefully kept simple to isolate the contribution of the various model features in explaining our results. While loan creation was endogenously created through loan market participation decisions, the destruction of credit relationships was exogenous in the basic model. An extended version is considered where the phenomena of credit break-ups is endogenized by introducing agency costs and the possibility of default. Such a model would then be able to address the cyclical behavior of loan destruction over the business cycle and provide an additional mechanism to propagate monetary shocks. Another natural extension which would make the model more conducive to business cycle analysis is the incorporation of productive capital. While our model is designed to study the impact of monetary shocks such an extension would allow it to consider how capital market search can generate a financial accelerator...
whereby real shocks to both aggregate and financial market productivity can be propagated. Finally, Chang and Li (2004) and Dressler and Li (2009) demonstrate that liquidity effects with the explicit production of credit services may deliver a monetary explanation for the lead-lag relationship between household and business investment over the cycle. Given that search frictions in household credit markets, such as mortgages, are likely to be very different than those in business credit markets, incorporating a household lending channel into this framework might allow a more comprehensive analysis of investment dynamics in response to persistent liquidity effects.
References


FIGURE 1A - Steady State Money Growth & Inflation

($\psi = 0.1$ and $\psi = 1.0$)
FIGURE 1B - Steady State Money Growth & Inflation
($\psi = 0.1$ and $\psi = 1.0$)
FIGURE 2A - NLP \((\rho = 0 \text{ and } \psi = 0.10)\)
FIGURE 2B - NLP \((\rho = 0 \text{ and } \psi = 0.10)\)
FIGURE 3A - BNLP and NLP ($\rho = 0.32$ and $\psi = 1$)
FIGURE 3B - BNLP and NLP ($\rho = 0.32$ and $\psi = 1$)
FIGURE 4A - NLP ($\rho = 0.32$ and $\psi = 0.10, \psi = 1$)
FIGURE 4B - NLP \((\rho = 0.32 \text{ and } \psi = 0.10, \psi = 1)\)
FIGURE 5A - BLP and LP ($\rho = 0.32$ and $\psi = 1$)
FIGURE 5B - BLP and LP ($\rho = 0.32$ and $\psi = 1$)
FIGURE 6A - LP ($\psi = 0.10$ and $\psi = 1$)
FIGURE 6B - LP ($\psi = 0.10$ and $\psi = 1$)
FIGURE 7A - LSC ($\alpha = 0.005$) and HSC ($\alpha = 0.05$)
Figure 7B - LSC ($\alpha = 0.005$) and HSC ($\alpha = 0.05$)