Income Inequality and Sovereign Default*

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Abstract

In this paper, we study the role of income inequality in government's borrowing and default decisions. We consider a standard endogenous sovereign debt default model and extend it to allow for heterogeneous agents. In addition to shocks to the average income level, we consider the effect of shocks to income distribution. Consistent with the data, income dispersion across individuals increases during a recession and decreases during an expansion. The model is calibrated to match a number of stylized facts for Argentina. We show that (i) rising income inequality within a country increases the probability of default significantly; (ii) the effect of output shocks is larger than the effect of inequality shocks; (iii) the joint effect of these two shocks can generate a high default probability consistent with the Argentine data; (iv) the model can match the high volatility of consumption of the poor relative to the rich; (v) progressive income taxes can reduce the default risk.

Keywords: Sovereign debt; Default; Income Inequality; Redistribution.

JEL codes: F3, F4, E5, D5

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1 Introduction

Emerging markets tend to experience high rates of sovereign default and they are also characterized by high income inequality that changes over time. In sovereign default literature, the line of research following Eaton and Gersovitz (1981) has focused mainly on the effects of output shocks on the government's borrowing and default decisions. In this paper, we study the role of income inequality in government's borrowing and default decisions. Does higher income inequality increase the probability of default? Furthermore, how do changes in inequality compare to changes in output in explaining the variation in default probability?

In order to explore the relationship between endogenous default risk and income inequality, we consider a stochastic general equilibrium model following an approach similar to that of Eaton and Gersovitz (1981). We model a small open economy with two types of households. In addition to output shocks that affect the average level of endowment, we introduce shocks that affect its distribution, which we call inequality shocks.[1] The economy is subject to aggregate uncertainty about future endowments, and households cannot completely insure against the shocks. The output and inequality shocks have different effects on the endowments; an adverse output shock lowers the endowments of both types, but an adverse inequality shock raises the endowment of the rich households and reduces the endowment of the poor households, increasing the dispersion between the endowments. There is also a benevolent government that represents the preferences of the households and can issue non-state-contingent, one-period bond contracts to borrow from risk-neutral foreign lenders, retaining the option to default at any time. We assume that default entails exogenous drops in output and that the economy goes into autarky temporarily. The government internalizes how its borrowing decisions affect the default risk, as well as the price of bonds, which determines the interest rates.

In our model, the government would like to borrow on behalf of households for two reasons. First, the government uses bond contracts and rebates the proceeds of debt operations equally across households to help them smooth consumption. Second, the equilibrium interest rate is lower than the inverse of the discount rate of the government, so the government would like to shift future consumption to today by borrowing. The level of existing debt and the size of the shocks are crucial for government's borrowing decision. As the

[1] Even though our model treats the changes in income inequality as exogenous, these shocks can be motivated by the fact that idiosyncratic labor earnings risk exhibits countercyclical volatility, as shown by Storesletten et al. (2004).

debt accumulates, it becomes harder to roll over because the benefits of borrowing diminish. Defaults are particularly more attractive in recessions, in high inequality states and when there is high debt accumulation because foreign lenders offer bond contracts that have higher interest rates in those states, which creates a borrowing constraint for the government. The government's goal is to maximize household's expected lifetime utilities, so it achieves this goal by trying to equalize the marginal utilities of consumption between households and across time. Default can reduce the gap in the marginal utilities of consumption between the two types of households because the burden of debt payment can be eliminated. Consequently, in our model, default can serve as a redistribution mechanism that improves households' welfare. The main finding of this paper is that inequality shocks can increase the default risk significantly. The key intuition for this result is that when the economy is subject to both adverse inequality and output shocks, the marginal utility of consumption of the poor increases significantly relative to the marginal utility of consumption of the rich. This generates a large tax burden, particularly on poor households, and the government chooses to default more often to wipe out the debt burden.

The model is calibrated using quarterly Argentine data between 1990-2002 similar to Arellano (2008). When we consider the role of each shock in Argentina, we find that default risk is slightly higher when there are output shocks than inequality shocks. This is because the implied default penalties are different in the two models. In the case of output shocks, the default penalty is higher in good states of the world and smaller otherwise. So with smaller penalty and tighter borrowing constraints in bad states, we observe a larger default rate. On the other hand, in case of inequality shocks the default penalty is constant across all states because aggregate output is constant. This generates a lower probability of default in this model. However, each shock alone can generate only about one sixth of the probability of default observed when there are both shocks in the model. Thus, we show that the joint effect of these shocks helps the model generate a default probability consistent with the data. The reason behind this result is the VAR(1) process estimated from the Argentine data. Based on the estimates of the structural parameters, we find that high inequality at time $t-1$ leads to lower output at time t. Also, the estimates of the covariances of the shocks are negative, which implies that there is more likely to be an adverse output shock together with an adverse inequality shock. These characteristics play an important role in lenders' and the government's expectations about the future state of the economy. An adverse inequality shock not only amplifies the effect of a low output shock today, but also creates a deep-seated pessimism that the recession will be more severe in the future. As a result, foreign lenders ask for a higher premium even for smaller levels of debt. This increases the borrowing constraints for the government, and default becomes the optimal decision.

We simulate the model to generate the business-cycle statistics. Our model's results regarding the default probabilities can be compared to the results in Aguiar and Gopinath (2006).[2] Similar to Aguiar and Gopinath (2006), the default probability when the economy is hit by an output shock is quite low, only 0.52 percent. Aguiar and Gopinath (2006) also use shocks to the trend of output and generate a default probability of around two percent. On the other hand, the inequality shocks generate a default probability of 0.32 percent. Using shocks to both output and inequality, our model can match countercyclical interest rates, high volatility of consumption and output, and a positive correlation between the trade balance and interest rate spreads. In addition to that, inequality is countercyclical with output and positively correlated with interest rate spreads.

An important contribution of the paper that has not been shown by the existing papers in the literature before is that the model can also generate high consumption volatility of the poor households relative to the rich households. We find that the ratio of the volatilities is close to its data counterpart for Mexico. Incomplete assets together with output and inequality shocks are key for this result. In the model, income inequality shocks amplify the effect of output shocks particularly on the poor households' endowment. Since there are no other assets that the households can use to insure against these shocks, poor households have higher volatility of consumption than the rich households.

As a policy exercise, we extend the model by introducing progressive income taxes and analyze the effect of these taxes on the debt levels and the default probability. When it is costly to borrow for the government, i.e. the proceeds of the debt operations are negative, the government finances the existing debt by issuing progressive income taxes. We adopt the progressive tax regime that Heathcote et al. (2014) present. However, when it is cheap to borrow, the government does not tax households, it simply distributes the transfers across households. We show that as the progressivity of the tax increases, the probability of default decreases. We show that if Argentina imposed a progressive tax regime that is as progressive as the one in Germany, the default risk would decline by about 60 percent. The tax system helps eliminate the effect of inequality shocks in the model and reduces the dispersion in the marginal utilities of consumption between households. Therefore, we obtain larger debt in the simulated economies.

[2] Aguiar and Gopinath (2006) assume a representative agent model; and their default penalty structure and calibration strategy are different than ours.

This paper relates to the recent quantitative models that explore emerging markets' business cycles and sovereign debt. We contribute to the sovereign default literature by incorporating the role of income inequality as an additional source of default risk. The endogenous sovereign default literature starts with the seminal paper of Eaton and Gersovitz (1981) and continues with Aguiar and Gopinath (2006), Arellano (2008), Broner et al. (2010), Yue (2010), Pitchford and Wright (2011), Chatterjee and Eyigungor (2013), Aguiar and Amador (2013b) and Gennaioli et al. (2014), some of which were mentioned above.[3] Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012), and Chatterjee and Eyigungor (2012) consider long maturity bonds in a representative agent framework. Cuadra and Sapriza (2008) and Hatchondo et al. (2009) study the role of political uncertainties in sovereign default risk. Broner et al. (2010) and Broner et al. (2008) show that well-functioning secondary markets can eliminate the default risk. All these papers use representative agent models and focus on the role of output shocks. Our paper is also closely related to D'Erasmo and Mendoza (2014) and D'Erasmo and Mendoza (2016), the main focus of which is the relationship between wealth inequality and default using a heterogeneous agent framework. D'Erasmo and Mendoza (2014) have endogenous wealth heterogeneity that comes from idiosyncratic income shocks; however, the amount of bonds is determined by a fiscal reaction function and does not come from the maximization of household utility. As mentioned above, in our model, the government optimally chooses the level of next-period bonds taking into account the welfare of the households. Furthermore, we show that income inequality shocks tend to have a systematic relationship with output shocks, so we incorporate this dimension into our model to generate inequality. D'Erasmo and Mendoza (2016) study the distributional effects of sovereign debt default in a two-period, closed economy model, assuming an exogenous initial wealth distribution. In their closed economy setup, they study optimal debt and default decisions on domestic debt. However, in our model, we focus on borrowing and default on external debt in a small open economy. In this sense, our paper is complementary to these two papers. Cuadra et al. (2010) study fiscal policy and default risk using a representative agent model, in which tax on consumption is endogenously determined and the revenues are used to finance public goods. In our paper, we assume progressive taxes on income.

Our paper is also related to the immense empirical literature that studies the determinants of sovereign default. Cantor and Packer (1996) show that income, external debt and economic development are significant determinants of credit risk. Reinhart et al. (2003)

[3] Also see Panizza et al. (2009), Wright (2011) and Aguiar and Amador (2013a) for good reviews of this literature.

show that a country's past behavior about meeting its debt obligations can be a good predictor of its ability to pay future debt, pointing out the importance of financial institutions. Hatchondo et al. (2007) argue that countries are more likely to default during periods with low resources, high borrowing costs and changes in political circumstances, and González-Rozada and Yeyati (2008) examine the role of global factors, such as liquidity, risk appetite and contagion, in explaining the emerging market spreads.

The rest of the paper is organized as follows: We provide a more formal analysis of the empirical results, showing the relationship between income inequality and credit scores in Section 2. We then present the model and define the recursive equilibrium in Section 3. We discuss the calibration, the quantitative analysis of the model and the simulation results with counterfactual experiments in Section 4. Section 5 discusses the differences in consumption volatilities between rich and poor households. Section 6 presents the effects of progressive income taxes. Section 7 concludes.

2 Empirical Motivation

In this section, we provide empirical results that support the relationship between income inequality and default risk. We use credit ratings dataset as a measure of default risk. Reinhart (2002) shows that credit ratings can predict defaults well.[4] We show that income inequality is positively correlated with the creditworthiness of sovereign bonds.

2.1 Income Inequality and Credit Ratings

Reinhart et al. (2003) show that there is a strong relationship between external debt and credit ratings. In order to present some empirical evidence for the effect of inequality on the credit worthiness of sovereign bonds, we follow an approach similar to that in Cantor and Packer (1996)[5]. We use the following specification to estimate the effect of inequality on credit scores in our benchmark estimation:

$$\begin{aligned} \textit{Credit Score}_{i,t} &= \alpha_0 + \alpha_1 \textit{Gini}_{i,t-1} + \alpha_2 \textit{Debt-to-GDP}_{i,t-1} \\ &+ \alpha_3 \textit{GDP per capita}_{i,t-1} + u_i + z_t + \textit{error}_{i,t} \end{aligned} \tag{1}$$

[4]They show that this relationship is robust using various credit-score datasets such as Institutional Investor ratings, Standard and Poor's and Moody's.

[5]Our analysis relies on a panel estimation, whereas Cantor and Packer (1996) perform a cross-sectional analysis.

To measure the creditworthiness of sovereign bonds, we use the Fitch credit ratings data for long-term bonds that are issued under foreign currency and obtain them from Bloomberg. This dataset covers a period between 1994 and 2012. For income inequality, we use the market (pre-tax, pre-transfer) Gini indices provided by the Standardized World Income Inequality Database (SWIID) (Solt, 2016). This is an unbalanced panel dataset that has information on inequality for 167 countries covering 1960 to 2013. Debt-to-GDP ratio is the external debt-to-GDP ratio from the Reinhart-Rogoff series that extends until 2010. Most of this dataset comes from IMF's Standard Data Dissemination Service, and it is defined as the outstanding amount of those actual current liabilities that require payments of principal and/or interest that residents of an economy owe to non-residents (TFFS, 2013). The real GDP per capita series is from the World Bank, and we take its log for the estimations. GDP series is PPP adjusted and measured in constant 2011 international dollars. In order to perform a regression using the credit ratings, we assign a numerical value similar to that in Cantor and Packer (1996) and Reinhart (2002). Table 13 shows the conversion of the ratings to scores in Appendix.

We expect to obtain a negative coefficient on Gini and debt-to-GDP ratio and a positive coefficient on GDP per capita. This implies that higher inequality in country i at time $t-1$ reduces the credit score in the next period. The credit score of a country shows how risky that country's bond is, and higher inequality increases the riskiness, which is reflected by a lower credit score.

Table 1 shows the summary statistics for the variables used for the regression sample, which covers the period 1994-2010 and contains 56 countries. A couple of differences stand out when we compare observations of emerging markets and advanced economies. First, emerging markets have low ratings even though their debt-to-GDP ratios are not very high, which can be attributed to debt-intolerance as Reinhart et al. (2003) put it. Second, they also have higher income inequality and lower GDP per capita than advanced economies have.

We estimate equation 1 using year (z_t) and country (u_i) fixed effects. We are interested in analyzing the effect of inequality that varies over time; therefore, country fixed effects will control for time-invariant characteristics unique to a country. In the first specification, we find that an increase in country's external debt-to-GDP ratio is associated with lower credit worthiness in the next period. This is a standard result in the literature, as well. We also find that an increase in income is associated with an increase in country's credit worthiness. Finally, an increase in Gini index is negatively associated with the credit worthiness in the

Table 1: Country Ratings, Debt, Income Inequality and GDP per Capita

Country	Average Fitch rating	Average external debt/GDP	Average inequality	Average GDP per capita
Emerging market economies				
Argentina	CCC/CCC+	46.44	47.05	4,413
Bolivia	B-/B	65.94	55.05	4,609
Brazil	BB-/BB	26.96	57.52	10,200
Bulgaria	BB/BB+	105.53	34.48	12,036
Chile	A-/A	41.24	54.22	15,344
China	A-/A	14.39	46.78	5,420
Colombia	BB+/BBB-	30.74	53.48	9,193
Costa Rica	BB/BB+	32.58	49.07	10,356
Dominican Rep.	B-/B	27.69	53.09	8,148
Ecuador	CCC/CCC+	68.28	53.01	8,342
Egypt, Arab Rep.	BB+/BBB-	40.17	36.79	7,863
El Salvador	BB/BB+	38.02	48.92	6,483
Ghana	B/B+	73.68	41.66	2,368
India	BB+/BBB-	23.77	50.24	2,925
Indonesia	B+/BB-	65.54	41.08	6,809
Kenya	B+	84.66	53.32	2,271
Malaysia	BBB/BBB+	42.93	48.10	16,943
Mexico	BB+/BBB-	30.73	48.70	14,466
Nigeria	BB-	73.12	51.56	3,051
Panama	BB+/BBB-	67.22	53.85	11,342
Peru	BB/BB+	46.29	52.66	7,359
Philippines	BB/BB+	59.93	50.50	4,602
Romania	BB/BB+	33.54	39.30	12,952
Russia	BB +/BBB-	40.06	49.46	16,665
South Africa	BBB-/BBB	23.37	72.11	10,717
Sri Lanka	CCC+/B-	55.24	45.00	6,227
Thailand	BB+/BBB-	47.72	47.22	10,342
Tunisia	BBB-/BBB	62.44	43.60	7,993
Turkey	B+/BB-	41.62	48.81	13,886
Uruguay	BB/BB+	38.73	51.27	13,488
Venezuela	B+/BB-	43.27	43.83	15,765
Advanced economies				
Australia	AA/AA+	62.58	47.70	35,546
Austria	AAA	128.72	44.98	38,639
Belgium	AA/AA+	203.15	47.07	36,558
Canada	AA/AA+	64.42	46.35	36,349
Denmark	AA+	119.77	45.16	40,506
Finland	AA/AA+	97.55	47.34	34,325
France	AAA	105.64	49.57	34,102
Germany	AAA	94.92	49.06	37,104
Greece	BBB/BBB+	83.37	48.85	26,027
Hungary	BBB/BBB +	81.31	50.63	19,024
Iceland	A-/A	286.19	36.69	34,789
Ireland	A+/AA-	441.11	50.78	37,327
Italy	A+	77.77	48.47	34,761
Japan	AA/AA+	34.62	43.68	32,700
Korea, Rep.	A-/A	27.07	35.15	22,759
Netherlands	AA-/AA	179.54	46.38	40,793
New Zealand	AA/AA+	72.39	49.20	28,753
Norway	AAA	77.44	42.97	57,545
Poland	BBB/BBB+	45.39	48.20	15,868
Portugal	AA-/AA	123.33	51.35	24,933
Singapore	AA+	129.31	44.19	55,998
Spain	AA/AA+	81.50	49.48	29,950
Sweden	AA-	114.68	47.30	36,973
Switzerland	AAA	190.81	40.83	49,406
United States	AAA	57.36	49.11	45,040

List of countries used in the panel regression. Time period covers 1994-2010. Data sources from left to right: Bloomberg L.P., Reinhart-Rogoff series, SWIID version 5.0, the World Development Indicators database.

Table 2: Panel Regressions Explaining Credit Worthiness

Independent Variable	Dependent Variable: Score of country i in year t. (1)	(2)	(3)	(4)
External debt-to-GDP at $t-1$	-0.0085^{***}	-0.0088^{*}	-0.0154^{***}	-0.0080^{***}
	(0.0011)	(0.0011)	(0.0077)	(0.0012)
GDP per capita at $t-1$	8.5140^{***}	8.1386^{***}	8.5624^{**}	5.6706^{***}
	(2.2530)	(2.1715)	(3.2164)	(2.0694)
Gini at $t-1$	-0.0593^{*}	-0.0582^{*}	-0.0647^{**}	-0.1131^{*}
	(0.0344)	(0.0327)	(0.0313)	(0.0580)
GDP growth at $t-1$	-	0.0001	0.0078	0.1034^{**}
		(0.0312)	(0.0302)	(0.0449)
Inflation rate at $t-1$	-	-0.0006^{***}	0.0001	-0.0120
		(0.0002)	(0.0002)	(0.097)
Current account-to-GDP at $t-1$	-	-0.0625^{***}	-0.0023^{*}	-0.0136
		(0.0289)	(0.0293)	(0.0241)
Time period	1994-2010	1994-2010	1994-2001	2002-2010
Year fixed effects	Yes	Yes	Yes	Yes
Country fixed effects	Yes	Yes	Yes	Yes
No of countries	56	56	46	47
N	546	546	275	271

The dependent variable is the credit score of country i in year t. Estimation is by robust standard errors. Standard errors are reported in parentheses. Per capita GDP is in logs. (***,**,* represent significance level at 1%, 5% and 10%, respectively.)

next period. The estimate is significant at ten percent and robust to country and time fixed effects.

The literature has also focused on other determinants of default risk, such as GDP growth rate, inflation rate or current account-to-GDP. Therefore, in the second specification we introduce these variables in order to see whether our result pertaining to the Gini index is robust to other specifications. GDP growth rate, inflation rate based on CPI series and current account-to-GDP series are obtained from the World Bank. Unlike the findings by Cantor and Packer (1996), the coefficient of GDP growth rate is insignificant. However, it is positive, which supports the fact that countries have easier time to service the debt burden when their economies expand. We find that higher inflation rate reduces the creditworthiness in the next period because usually high inflation rate is linked to structural issues in the economy, such as government's incentive to resort to inflationary debt finance (Cantor and Packer, 1996). We obtain a negative coefficient of current account. This might be explained by the fact that current account deficits have increased in advanced economies (Blanchard, 2007), yet current account deficits may not necessarily make debt service harder over time. Next, in the third and fourth specifications we divide the data into two time periods. Reinhart and Rogoff (2009) show that in the 1990s and early 2000s the share of countries in external default or restructuring weighted by their share of world income was around 5-10 percent and it plummeted by the end of 2001. Therefore, as a robustness check we consider two periods, 1994-2001 and 2002-2010.[6] We find that the coefficient of Gini index is still negative and becomes more significant between 1994-2001.

3 Model

In this section, we present a model economy in order to structurally analyze the role of inequality in sovereign debt default. Our model is similar to the model presented by Arellano (2008) and belongs to the class of models in the standard framework of Eaton and Gersovitz (1981). We consider a discrete time, small open economy inhabited by heterogeneous agents that are hand-to-mouth and differ in the stochastic endowments they receive. The endowment is subject to aggregate output and inequality shocks that cannot be completely insured against. There is a benevolent government that represents the preferences of households and has access to international markets. The government can issue one-period bonds to foreign lenders and rebate the proceeds of the debt operations to the households. The government can choose to default fully on its debt at any time, because contracts are not enforceable. The penalty for default is that the economy is forced into financial autarky for a period of time, and there is an exogenous drop in output. Now, we move on to the details of the model.

3.1 Households

There are two types of infinitely-lived households indexed by $i = 1, 2$, and their preferences over consumption of the good, c_t, is assumed to be

$$u(c_t^i) = \frac{c_t^{i,1-\sigma}}{1-\sigma} \tag{2}$$

where σ is the constant relative risk-aversion parameter, and $\sigma > 0$ and $\sigma \neq 1$. The type 1 household receives a stochastic stream of a tradable good, $\frac{(1+\gamma)y}{2}$, and type 2 receives $\frac{(1-\gamma)y}{2}$, where y and γ denote output and inequality, respectively. The output y and the inequality

[6] Since, we do not have recent data on credit ratings our sample is unable to cover the European debt crisis. Our estimation is also robust to choosing 2002-2003 as a cutoff point. Results are available upon request.

γ follow a Markov process with a transition function $f(y', \gamma' | y, \gamma)$. Households also receive an equal amount of transfer from (or pay taxes on goods to) the benevolent government in a lump sum fashion. Households live hand-to-mouth, which means they do not make any individual saving or borrowing decisions.

3.2 Government

The government of the economy can trade one-period, non-state contingent bonds with foreign lenders that are risk neutral and competitive. As in a standard default model, when the government defaults, the economy faces two types of exogenous default penalties: direct output costs and a temporary exclusion from borrowing in the debt markets. The government's goal is to maximize social utility, which is the expected discounted sum of lifetime utilities of both types with equal weights given as

$$E_0 \sum_{t=0}^{\infty} \beta^t \Big[u(c_t^1) + u(c_t^2) \Big], \tag{3}$$

where β denotes the discount parameter and $\beta \in (0, 1)$. The government makes two decisions in this model. First, it decides whether to repay or default on its existing debt. Second, conditional on not defaulting, it chooses the amount of one-period bonds, B', to issue or buy. If the government chooses to buy bonds, the price it needs to pay is given as $q(B', y, \gamma)$. The discount bonds, B', can take a positive or negative value. If it is negative, this means that the government borrows $-q(B', y, \gamma)B'$ amounts of period t goods and promises to pay B' units of goods in the next period, if it does not default. Similarly, if B' is positive, then this implies that the government saves $q(B', y, \gamma)B'$ amounts of period t goods and will receive B' units of goods in the next period. The bond price function $q(B', y, \gamma)$ depends on the size of the bonds, B', income shock, y, and inequality shock, γ. Government internalizes how its borrowing decisions affect the default risk and the price of the bond.

When the government chooses to repay its debt, the resource constraint for household 1 is

$$c^1 = \frac{(1 + \gamma)y}{2} + \frac{B - q(B', y, \gamma)B'}{2}, \tag{4}$$

and the resource constraint for household 2 is

$$c^2 = \frac{(1-\gamma)y}{2} + \frac{B - q(B', y, \gamma)B'}{2}. \tag{5}$$

The economy faces three types of uncertainty that cannot be insured away with non-state-contingent bonds. The first one is the dispersion in incomes induced by shocks to γ. The second one is the output shock y that affects the aggregate output in the economy. Finally, the third one is the endogenous default risk. The goal of the government is to maximize the expected utilities of households, and it achieves this goal by trying to equalize the marginal utilities of consumption between households and across time. One government policy is to choose optimal B' that satisfies its goal, and the level of existing debt and the size of the shocks are crucial for this decision. As debt accumulates, it becomes harder to roll it over because of increasing default risk.

When the government chooses to default, consumption of the types are:

$$c^1 = \frac{1+\gamma}{2} y^d \tag{6}$$

and

$$c^2 = \frac{1-\gamma}{2} y^d, \tag{7}$$

where y^d is the level of output in default and $y^d = y - \kappa(y)$. The penalty is a function of the output such that $\kappa(y) = \min\{y, \max\{0, d_0 y + d_1 y^2\}\}$. The default penalty is higher, if default happens in the good states of the world. This default penalty structure has been used in many papers in the literature, such as Chatterjee and Eyigungor (2012).

3.3 Foreign Creditors

Foreign creditors can perfectly monitor the state of the economy and have perfect information about the shock processes. They can borrow loans from international credit markets at a constant interest rate $r > 0$, which is the risk-free interest rate for this model. Taking the bond price function $q(B', y', \gamma)$ as given, they choose loans B' that maximize their expected profits ϕ, given as

$$\phi = q(B', y, \gamma)B' - \frac{1 - \delta(B', y, \gamma)}{1+r} B', \tag{8}$$

where $\delta(B', y, \gamma)$ is the probability of default and it is determined endogenously.

$$q(B', y, \gamma) \quad = \quad \begin{cases} \frac{1}{1+r} & B' \geq 0 \\ \frac{1-\delta(B', y, \gamma)}{1+r} & B' < 0. \end{cases}$$

The price function depends on the sign of B'. It is never optimal to default when the government saves ($B' \geq 0$), so in that case, the price is a constant function of the risk-free interest rate. On the other hand, if the government borrows ($B' < 0$), then the price reflects the default probability. This implies that as the default probability increases, the price of the bond falls.

3.4 Timing

The timing in the model is as follows.

1. The government starts with initial assets B.
2. The output shock y and the inequality shock γ are realized.
3. The government decides whether to repay its debt obligations or default.
   (a) If the government decides to repay, then taking as given the bond price schedule $q(B', y, \gamma)$, the government chooses B' subject to the resource constraint. Then creditors, taking $q(B', y, \gamma)$ as given, choose B'. Finally, households consume c^1 and c^2 with respect to their types.
   (b) If the government chooses to default, then the economy is in financial autarky and remains in autarky in the next period with probability θ. Households simply consume their endowments.

3.5 Recursive Equilibrium

We focus on a recursive equilibrium, in which there is no enforcement. Based on the foreign creditors' problem, government's debt demand is met as long as the gross return on the bond equals $(1 + r)$. Given loan size B', inequality state γ and income state y, the bond price is

$$q(B', y, \gamma) = \frac{1 - \delta(B', y, \gamma)}{1 + r}. \tag{9}$$

The value function for the government that has the option to default or pay its debt is given as $v^o(B, y, \gamma)$. Government chooses the option that maximizes the welfare of agents. The default option will be optimal only if the government has debt. The value of default is denoted by the function $v^d(y, \gamma)$, and the value of repayment is denoted by $v^c(B, y, \gamma)$.

$$v^o(B, y, \gamma) = \max_{c,d}\{v^c(B, y, \gamma), v^d(y, \gamma)\}. \tag{10}$$

The value of default is expressed by

$$\begin{aligned} v^d(y, \gamma) &= u\left(\frac{(1+\gamma)y^{def}}{2}\right) + u\left(\frac{(1-\gamma)y^{def}}{2}\right) \\ &+ \beta \int_{\gamma'} [\theta v^o(0, y', \gamma') + (1-\theta)v^d(y', \gamma')] f(y', \gamma' | y, \gamma) d(\gamma', y'). \end{aligned} \tag{11}$$

Under default, individuals only consume their income. The government can gain access to debt markets with probability θ, and the economy stays in autarky with probability $1 - \theta$. The transition probabilities are given by the joint density function, f. Similarly, the value of staying in contract is

$$\begin{aligned} v^c(B, y, \gamma) &= \max_{B'} u\left(\frac{(1+\gamma)y - q(B', y, \gamma)B' + B}{2}\right) + u\left(\frac{(1-\gamma)y - q(B', y, \gamma)B' + B}{2}\right) \\ &+ \beta \int_{y',\gamma'} v^o(B', y', \gamma') f(y', \gamma' | y, \gamma) d(\gamma', y'). \end{aligned} \tag{12}$$

If the government chooses to repay its debt, the value function for this choice reflects the future options for default and staying in contract. The government chooses the optimal bond contract that maximizes the sum of utilities of the households and expected discounted future value of option.

We can characterize the government's default policy by default and repayment sets. Let $A(B)$ be the set of y and γ for which repayment is optimal when assets are B, such that

$$A(B) = \left\{(y, \gamma) \in (\mathbb{Y}, \Gamma) : v^c(B, y, \gamma) \geq v^d(y, \gamma)\right\}, \tag{13}$$

and let $D(B) = \tilde{A}(B)$ be the set of y, γ for which default is optimal for a level of assets B:

$$D(B) = \left\{(y, \gamma) \in (\mathbb{Y}, \Gamma) : v^c(B, y, \gamma) < v^d(y, \gamma)\right\}. \tag{14}$$

Proposition 1. *Given an output shock y, inequality shock γ and bond positions $B^1 < B^2 \leq 0$, if default is optimal for B^2 then default will be optimal for B^1, and the probability of default at equilibrium satisfies $\delta(B^1, y, \gamma) > \delta(B^2, y, \gamma)$.*

Proof. See Appendix. □

This proposition formally states a feature of the model that Eaton and Gersovitz (1981) also have. It shows that in equilibrium default sets expand and the probability of default increases as the level of debt in a country increases. The following proposition states that equilibrium bond price decreases as the level of debt increases.

Proposition 2. *Given an output shock y, inequality shock γ and bond positions $B^1 < B^2 \leq 0$, equilibrium bond price satisfies $q(B^1, y, \gamma) \leq q(B^2, y, \gamma)$.*

Proof. See Appendix. □

Now we define the recursive equilibrium for this economy. Let $s = \{B, y, \gamma\}$ be the set of aggregate states for the economy.

Definition 1. *The recursive equilibrium for this economy is defined as a set of policy functions for (i) consumptions $c^1(s)$, $c^2(s)$; (ii) government's asset holdings $B'(s)$, repayment sets $A(B)$, and default sets $D(B)$; and (iii) the price function for bonds $q(B', y, \gamma)$ such that:*

1. *Agents' consumption $c^1(s)$ and $c^2(s)$ satisfy the resource constraints, taking the government policies as given.*
2. *The government's policy functions $B'(s)$, repayment sets $A(B)$, and default sets $D(B)$ satisfy the government optimization problem, taking the bond price function $q(B', y, \gamma)$ as given.*
3. *Bonds prices $q(B', y, \gamma)$ reflect the government's default probabilities and default probabilities satisfy creditors' expected zero profits.*

In equilibrium, the bond price function should satisfy both the government's optimization problem and the expected zero profits in the lenders' problem. As mentioned, the probability of default endogenously affects the bond price. Using the default sets, we can express the probability of default such that:

$$\delta(B', y, \gamma) = \int_{D(B')} f(y', \gamma' | y, \gamma) d(y', \gamma'). \tag{15}$$

When default sets are empty, default is never optimal at the asset level B', so the probability of default equals zero, independent of the realized shock. When $D(B') = (\mathbb{Y}, \Gamma)$, government always chooses to default for all shock levels. Default sets are shrinking in assets.

4 Quantitative Analysis and Simulation

4.1 Quantitative Analysis

In this section, we describe the estimation procedure for the shock processes and then explain the calibration strategy. We use the model to analyze the debt dynamics in Argentina between 1990-2002, quantitatively. Focusing on an Argentine default episode enables us to compare our results with the ones in the existing literature.

4.1.1 Calibration and Functional Forms

We solve the model assuming that both output and inequality shocks are in play. We call this the benchmark model. In the benchmark model, output and inequality shocks are modeled as a VAR process. Next, in order to quantify the role of each shock and to assess the importance of the shocks in matching the high volatilities and particularly high default rates observed in emerging economies, we solve the model subject to only one shock at a time. Model II has only output shocks, and we assume that output follows an AR(1) process. Model III has only inequality shocks and, again, the inequality shock is modeled as an AR(1) process.

In the benchmark model, we assume that the VAR process for log output and inequality is as follows:

$$\begin{bmatrix} log(y_t) \\ \gamma_t \end{bmatrix} = \begin{bmatrix} c_y \\ c_\gamma \end{bmatrix} + \begin{bmatrix} \rho_{yy} & \rho_{y\gamma} \\ \rho_{\gamma y} & \rho_{\gamma\gamma} \end{bmatrix} \begin{bmatrix} log(y_{t-1}) \\ \gamma_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{\gamma t} \end{bmatrix},$$

where

$$\begin{aligned}
\begin{bmatrix} c_y \\ c_\gamma \end{bmatrix} &= \left[\boldsymbol{I} - \begin{bmatrix} \rho_{yy} & \rho_{y\gamma} \\ \rho_{\gamma y} & \rho_{\gamma\gamma} \end{bmatrix} \right] \begin{bmatrix} \mu_y \\ \mu_\gamma \end{bmatrix} \\
\boldsymbol{\varepsilon} &= \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{\gamma t} \end{bmatrix} \\
E\left[\boldsymbol{\varepsilon}\right] = \mathbf{0} \quad &\text{and} \quad Var\left[\boldsymbol{\varepsilon}\right] = \begin{bmatrix} \sigma_y^2 & \sigma_{y\gamma} \\ \sigma_{\gamma y} & \sigma_\gamma^2 \end{bmatrix}.
\end{aligned}$$

The estimated values are derived from Argentina's GDP and income inequality data between 1991Q1 and 2005Q2. We use real output in quarterly, seasonally adjusted, real series and covering the period 1993Q1 to 2001Q4 from the dataset in Arellano (2008).[7] We take logs of GDP and detrend these series using an HP filter. The data pertaining to inequality are constructed using the distribution of income series in World Development Indicators provided by the World Bank. We choose the same period as for GDP. In order to construct the inequality measure, we compute the total income share of the upper 50th percentile and lower 50th percentile. Then, we take the difference of the income shares and divide it by two, which gives us the dispersion from the mean income. Since only annual data are available, we adopt the Boots-Feibes-Lisman method to disaggregate the annual data into quarterly data. Both output and inequality shocks are then discretized into a 21-state Markov chain, using Tauchen (1986).

The discount factor β, and default penalty parameters d_0 and d_1 are jointly calibrated to target a default probability of 3 percent, debt-to-GDP ratio of 5.53 percent and mean spread of 6.23. We set the probability of reentry to 0.25, which implies it takes a year to gain access to bond markets.[8]

Table 3 shows the parameters that we use for the benchmark model's calibration. We set the risk-free interest rate to 1.7 percent to match the US five-year Treasury bond

[7] Arellano (2008) uses the data provided by the Ministery of Finance of Argentina.

[8] The calibrated value of β and the value of θ are close to the values used in the default literature. For instance, Yue (2010) assumes that $\beta = 0.72$, and Aguiar and Gopinath (2006) assume that $\beta = 0.925$. The value of parameter θ implies that, on average, autarky takes four quarters, assuming that the distribution of default lengths is exponential (Tomz and Wright (2007) and Pitchford and Wright (2011)). Dias and Richmond (2009) empirically show that it takes 5.7 years, on average, for countries to regain partial access to international capital markets and Gelos et al. (2011) document that average exclusion from the international markets declined to two years in the 1990s; however, endogenous sovereign default models with exogenous entry to the debt markets calibrate the parameter θ around 0.25. (Arellano (2008) chooses 0.282 and Aguiar and Gopinath (2006) choose 0.10).

Table 3: A Priori Parameters for Model I

Name	Parameters	Description
Risk-free interest rate	$r = 1.7\%$	US 5-year bond quarterly yield
Risk aversion	$\sigma = 2$	
Probability of reentry	$\theta = 0.25$	
Stochastic structure	$\begin{bmatrix} \rho_{yy} & \rho_{y\gamma} \\ \rho_{\gamma y} & \rho_{\gamma\gamma} \end{bmatrix} = \begin{bmatrix} 0.95 & -0.38 \\ 0.00 & 0.95 \end{bmatrix}$	Argentina's GDP
	$\begin{bmatrix} \sigma_y^2 & \sigma_{y\gamma} \\ \sigma_{\gamma y} & \sigma_\gamma^2 \end{bmatrix} = \begin{bmatrix} 0.0003 & -0.0001 \\ -0.0001 & 0.0001 \end{bmatrix}$	and income inequality
	$\begin{bmatrix} c_y \\ c_\gamma \end{bmatrix} = \begin{bmatrix} 0.12 \\ 0.01 \end{bmatrix}$	

Table 4: Calibrated Parameters for Model I

Name	Parameters	Calibrated Parameter	Target	Value Target
Discount rate	β	0.925	Default probability	3 percent
Default penalty	d_0	-0.691	Debt service-to-gdp	5.45 percent
	d_1	0.095	Mean spread	6.23

quarterly yield. The risk-aversion parameter σ is set to 2, as it is standard in the macro literature. We also report the estimates of the parameters in the stochastic shock process. Note that the correlation of the output at t and the inequality at $t-1$, $\rho_{y\gamma}$, is negative. This means that high inequality generates low output in the next period. Similarly, since $\rho_{\gamma y}$ is equal to zero, the output in the previous quarter does not affect the inequality in the current period. This relationship between inequality and output is not unique to Argentina. We find that other frequently defaulting economies, such as Brazil, Costa Rica, Dominican Republic, Ecuador and Uruguay, also have similar results in terms of the signs of the estimates. These results are reported in Table 14 in the Appendix.

For Model II, we remove the stochastic inequality shocks by setting the level of inequality to the mean inequality up to the default episode. This corresponds to setting γ equal to 0.66. The stochastic process for output is assumed to be a log-normal AR(1) process such that

$$log(y_t) = \rho_y log(y_{t-1}) + \epsilon_{yt}, \tag{16}$$

where $E[\epsilon_{yt}] = 0$ and $E[\epsilon_{yt}^2] = \sigma_y^2$, which are estimated from Argentina's GDP. We again discretize the output process into a 21-state Markov chain using the Tauchen method. We

Table 5: A Priori Parameters for Model II

Name	Parameters	Description
Risk-free interest rate	$r = 1.7\%$	US 5-year bond quarterly yield
Risk aversion	$\sigma = 2$	
Discount rate	$\beta = 0.925$	
Default penalty	$d_0 = -0.691$	
	$d_1 = 0.095$	
Probability of reentry	$\theta = 0.25$	
Inequality	$\gamma = 0.66$	Mean income inequality in Argentina
Stochastic structure	$\rho_y = 0.9351$	Argentina's GDP
	$\sigma_y = 0.0190$	

Table 6: A Priori Parameters for Model III

Name	Parameters	Description
Risk-free interest rate	$r = 1.7\%$	US 5-year bond quarterly yield
Risk aversion	$\sigma = 2$	
Discount rate	$\beta = 0.925$	
Default penalty	$d_0 = -0.691$	
	$d_1 = 0.095$	
Probability of reentry	$\theta = 0.25$	
Stochastic structure	$\rho_\gamma = 0.9851$	Argentina's Inequality
	$\sigma_\gamma = 0.0037$	
	$\mu_\gamma = 0.38$	

keep all else the same as in the benchmark model. Table 5 presents the parameters for the second model.

Similarly, we need to estimate the stochastic inequality process for Model III. We estimate the following AR(1) process:

$$\gamma_t = (1 - \rho_\gamma)\mu_\gamma + \rho_\gamma \gamma_{t-1} + \epsilon_{\gamma t}, \tag{17}$$

where $E[\epsilon_{\gamma t}] = \mu_\gamma$ and $Var(\epsilon_{\gamma t}) = \sigma_\gamma^2$, which are estimated from Argentina's inequality data. As with Model III, we discretize the inequality process into a 21-state Markov chain using the Tauchen method. We keep all else the same as in benchmark model. The parameters for the third model are presented in Table 6.

4.2 Model Solution

In this section, we begin with the analysis of the benchmark model's results and then elaborate on the intuition behind the workings of the model. Our solution algorithm is given in the Appendix.

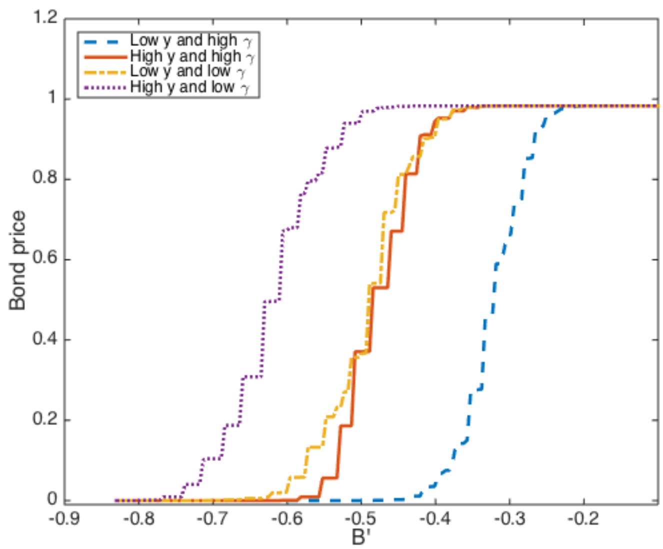
In our model, the benevolent government has two policy decisions to make: whether to repay the existing debt or default; and how much to borrow or save using one-period bonds. The government borrows to help households have smooth consumption and to shift future consumption to today because the equilibrium interest rate is lower than government's discount rate. The level of optimal debt depends on the current assets and the state of the world. Since lenders have full information about the state of the world and contracts are not state-dependent, borrowing constraints can bind for the government, particularly in bad states of the world, such as high inequality and low output. Therefore, we observe that bond prices depend on the level of assets and the types of shocks that the economy is subject to.

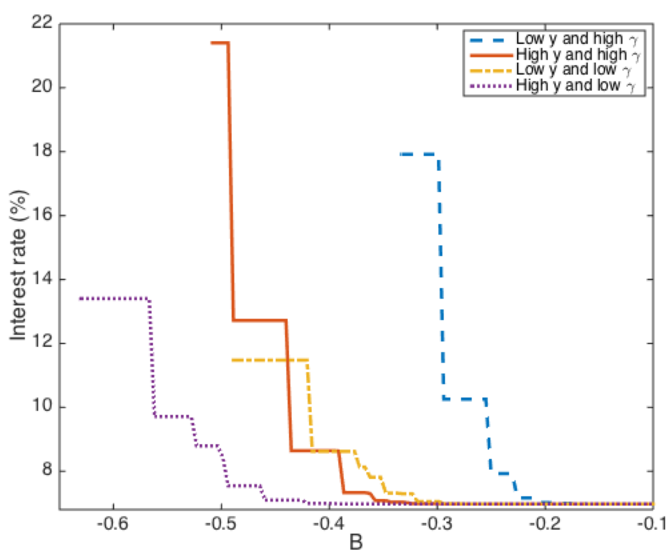
In the model, since the endowment is shared unequally among households, even in the absence of the shocks, the poor agents' marginal utility of consumption is higher than that of rich agents. An adverse output shock increases both agents' marginal utility of consumption, but an adverse inequality shock raises the marginal utility of the poor and reduces the marginal utility of the rich, increasing the dispersion between the marginal utilities of consumption. Defaults are more likely when there are adverse shocks and high levels of debt because the lenders offer bond contracts that have higher interest rates in these states. This makes the government borrowing-constrained and imposes large taxes on households in order to finance the debt. An adverse inequality shock exacerbates the burden of the tax, particularly on the poor, because it increases the poor's marginal utility of consumption disproportionately. In this case, the government can choose to default and use default as a redistribution mechanism. This policy improves welfare because, by eliminating the tax burden, the government can alleviate the dispersion.

First, we analyze our results related to policy functions and value functions in the benchmark model. We report the results based on four different combinations of output and inequality shocks. A low (high) shock is one standard deviation below (above) its mean for each type of shock. The level of assets is denoted as a fraction of GDP. Then, we look at the business-cycle statistics that the model generates.

Figure 1 shows the bond price schedule and the interest rate generated by the model.

Figure 1: Bond prices and interest rate (Model I)





On the x-axis we have assets as a fraction of output. Similar to the results presented in the standard default literature, such as Arellano (2008) and Aguiar and Gopinath (2006), we observe that bond prices are an increasing function of assets, such that high levels of debt entail a low bond price and a high interest rate. Fixing the level of inequality shocks, we observe that it is easier to borrow during expansions than during recessions. However, the results also show that the effect of a high output shock can be dominated by the effect of a high inequality shock. In other words, an economy that is subject to both high output and high inequality shocks can have a bond price that is lower than that when there are low output and low inequality shocks.

The lower panel in Figure 1 shows the annual equilibrium interest rates generated by the model. The interest rate is calculated as $1/q(B', y, \gamma) - 1$. Inequality shocks generate another source of risk that is reflected in interest rates. The highest level of borrowing is possible when there is high output and low inequality in the economy. Government borrowing is subject to higher interest rates, even for small amounts of debt that are above the level of default in high-inequality and or low-output states.

The top panel in Figure 2 shows the saving policy function conditional on not defaulting. Our results show that the government borrows more in expansions and when there is low inequality. This result is consistent with the countercyclical interest rates, since it becomes more costly to borrow in bad states of the world. The bottom panel of Figure 2 is the value function for the option to default or repay as a function of assets. Again, inequality plays a significant role in the default decision. The flat regions of the value function show the range of debt for which default is optimal. The value functions show that the highest debt can be supported, when there is high output and low inequality in the economy.

4.3 Business Cycle Results

4.3.1 Data

First, we document the business-cycle characteristics of the Argentine economy. For the business-cycle statistics, we use real output, consumption and trade balance data in quarterly, seasonally adjusted, real series for the period 1993Q1 and 2001Q4 from the dataset in Arellano (2008).[9] We take logs of GDP and consumption series and apply a linear trend on

[9] Arellano (2008) uses the data provided by the Ministery of Finance of Argentina.

Figure 2: Savings and value functions (Model I)

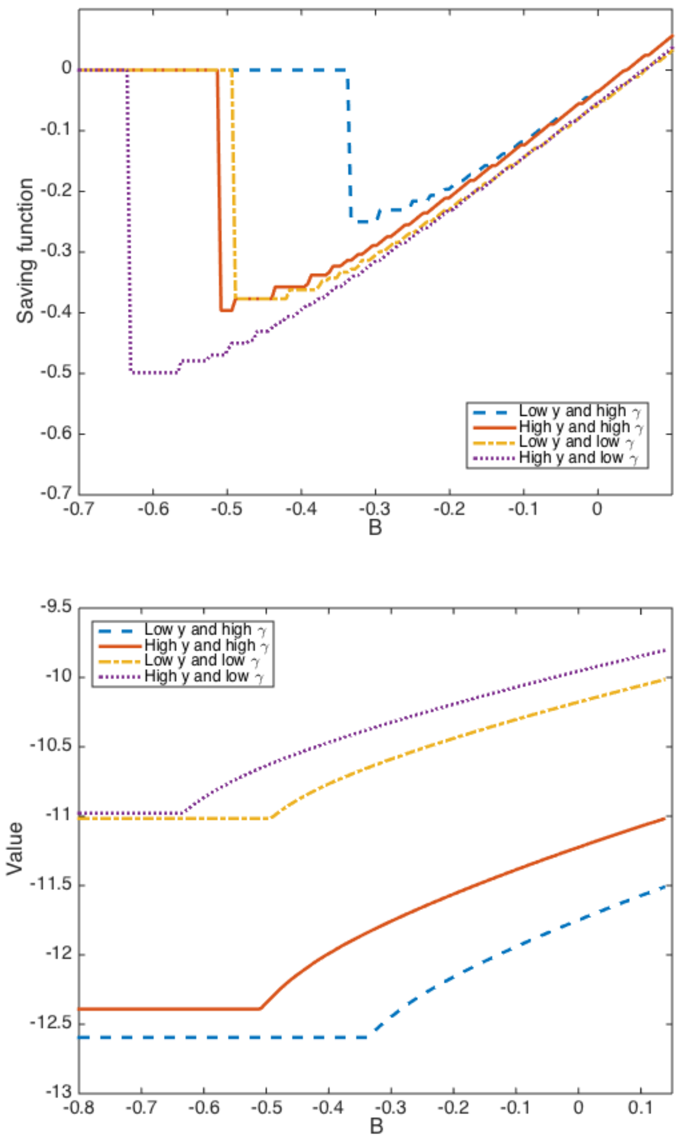


Table 7: Business Cycle Statistics for Argentina

	Default episode x: Q1-2002	std(x)	corr(x, y)	corr(x, r^c)
Interest rate spread (%)	28.60	2.77	-0.88	
Trade balance (% of GDP)	9.90	1.75	-0.64	0.70
Consumption (% deviation from trend)	-16.01	8.59	0.98	-0.89
Output (% deviation from trend)	-14.21	7.78		-0.88
Inequality (% deviation from mean ineq.)	8.6	1.71	-0.23	0.55

these series following Arellano (2008).[10] The trade balance data are a fraction of GDP. We also borrow Arellano (2008)'s spread data, which are defined as the difference between the interest rate in Argentina and the yield of the five-year U.S. treasury bond. The interest rate series is EMBI for Argentina and starts from 1983Q3. For the mean and standard deviation of the spread we use the period between 1993Q1 and 2001Q1. The inequality series is the one we constructed to generate a shock process, as explained in the previous section.

Table 7 presents the business-cycle statistics of all the data available up to the default episode that started on December 26, 2001. Consumption and output in the first column show the deviations from the trend, and the other values are in levels in the first quarter of 2002. Relative to the average inequality in the series, in the default episode, inequality increased by 8.6 percent. The second column shows the standard deviations up to the default episode. We find that consumption is more volatile than output. The third and the fourth columns present the correlations of each variable with the output and the interest rate spread, respectively. It has been shown that emerging market economies are characterized by countercyclical spread rates and net exports. Also, their consumption is highly correlated with output. We see similar empirical results for Argentina in column 3. In addition, we show that inequality is countercyclical with output, so the economy has high inequality during recessions. The interest rate spread is negatively correlated with consumption and output, and positively correlated with trade balance. The data show that inequality is positively correlated with the spread, which implies that inequality increases during times of risky borrowing.

[10] Analysis using HP filtered series (with smoothing parameter 1600) also produces similar results for correlations.

4.3.2 Simulation Results

Next, we move on to the business-cycle statistics generated by the benchmark model and evaluate the performance of the model with Argentine data. The upper panel of Table 8 presents the simulation results for the benchmark model, which generates a default probability of 2.80, debt-to-GDP ratio of 5.53 percent and mean spread of 4.90. High volatility of interest rates is a consequence of high default probability. We observe a large increase in the spread during default episodes, which is close to the data. In Argentina, in the couple of months following the default, quarterly spreads reached to 5,000-6,000 basis points. The model also generates large drops in consumption and output during default episodes. Inequality increases by 9.09 percent relative to its mean, which is also close to the increase observed during the default episode (8.60 percent). The model can also generate high volatility in consumption and output. The volatility of inequality is slightly lower than the value observed in the data.

Table 8: Simulation Results for the Benchmark Model

	Default episodes	std(x)	corr(x,y)	$corr(x, r^c)$
Model I: Shocks to output and inequality				
Interest rate spread (%)	59.82	9.94	−0.20	-
Trade balance (% of GDP)	−0.01	0.91	−0.12	0.29
Total Consumption (% deviation from trend)	−7.19	5.82	0.99	−0.25
Output (% deviation from trend)	−7.29	5.63	–	−0.20
Inequality (% deviation from mean ineq.)	7.45	0.70	−0.28	0.16
Other Statistics				
Mean debt (percent output)	5.53	Mean spread		4.90
Default probability	2.80			

In terms of correlations with output, the simulations can generate a positive correlation with consumption and a negative correlation with the interest rate spread.[11] We also obtain a negative correlation between output and trade balance. The reason is that when there are only output shocks, households can consume more than the level of the output during expansions because the government can borrow easily. On the other hand, when there is a recession, borrowing is constrained; therefore, the consumption is less than the output. This generates a countercyclical trade balance over the business cycle. We see a

[11] See Neumeyer and Perri (2005), Uribe and Yue (2006) and Aguiar and Gopinath (2007) for the role of countercyclical interest rates in emerging markets.

positive correlation between the spread and the trade balance. Since the spread reflects the risk due to both inequality and output shocks, it is more correlated with the bad states of the world, in which the government is more likely to face borrowing constraints and experience large trade balances. As we expected, inequality is negatively correlated with output and positively correlated with the spread.

We solve and simulate Model II and Model III, in order to assess the role of output shocks and inequality shocks in the default risk. The simulation results for Model II and Model III are given in Table 9. We find that the default probability is around 0.52 percent when there are output shocks and 0.32 percent when there are inequality shocks. We obtain a probability of default when the economy is subject to output shocks that is slightly higher than the model with inequality shocks because the default penalties are different in two models. In the case of output shocks, the default penalty increases in good states of the world and decreases in bad states of the world; thus with smaller penalty and tighter borrowing constraints in bad states, we observe a larger default rate. On the other hand, in model III the default penalty is constant across all states because aggregate output is constant. This generates a lower probability of default in model III.

Table 9: Simulation Results for Model II and Model III

	Model II		Model III	
	Default episodes	std(x)	Default episodes	std(x)
Interest rate spread (%)	9.70	1.46	1.78	0.68
Trade balance	−0.03	2.15	−0.02	0.98
Total Consumption	−2.78	5.65	−2.92	0.99
Output	−8.00	4.46	−7.99	0.00
Inequality	0.00	0.00	2.63	1.15
Other Statistics				
Mean debt (percent output)	52.76		48.46	
Default probability	0.52		0.32	
Mean Spread	0.63		0.44	

We also find that the default risk in both Model II and Model III is lower than that in the benchmark model. This is strong evidence that shows that the amplification effect comes from the underlying joint shock process. The reason behind this result is the VAR(1) process that we systematically estimated from the Argentine data. Based on the estimated process, it is more likely to have adverse output and inequality shocks together. Moreover, high inequality at time $t-1$ leads to lower output at time t. These characteristics play an important role in altering the expectations of foreign lenders and the government about the future state of the economy. An adverse inequality shock not only amplifies the effect of an adverse output shock today, but also generates pessimism that the recession with increasing inequality may be long-lasting.[12] As a result, foreign lenders ask for a higher premium, even for smaller levels of debt. This increases the borrowing constraints on the government, and default becomes an optimal decision.

5 Consumption Volatilities

Our goal in this exercise is to see whether the model can also explain differences in consumption volatilities across income groups that we observe in the data. Since we do not have the consumption distribution data for Argentina, we focus on Mexican economy for this exercise. Like Argentina, Mexico experienced several default episodes. We focus on the crises in the last century when we compute the default probability. According to Reinhart and Rogoff (2011), Mexico experienced external default or restructuring in 1914, 1928 and 1982, which leads to a default rate of 3 percent.[13] Two economies are similar in their business cycle statistics as shown in Table 10.[14]

We use Mexico Household Income and Expenditure Survey data between 1992 and 2008. We compute the consumption of the upper and lower 50 percentile of the households in order to make the statistics comparable with the model. In the data, we find that consumption volatility of the poor household is slightly higher than the rich household's and the ratio of volatilities is 1.10. Since survey data set is annual, using the simulated results and aggregating the data we convert the consumption of the poor and rich households to annual frequency. In our model, consumption volatility of the poor households is also

[12] In order to disentangle the effect of inequality on output in the next period, when we generate the Markov process, we assume that $\rho_{y\gamma} = 0$ and $\rho_{\gamma y} = 0$. Under this specification, we find that the probability of default falls to 1.96 percent. This result shows that two thirds of the default risk comes from the fact that the covariances of the shocks are negative.

[13] The country was at near default in 1994 according to Reinhart and Rogoff (2011), so we exclude this incidence when we compute the default rate.

[14] We borrow debt service-to-GDP statistic from Cuadra et al. (2010). Debt service to GDP data cover the years from 1980 to 2007 and the spread covers the period from 2000q1 to 2012q4. We compute the spread as the difference between the interest rates on government securities and treasury bills of Mexico and the U.S., both data are retrieved from FRED database provided by Federal Reserve Bank of St. Louis. We compute the business cycle statistics using quarterly seasonally adjusted real GDP, real consumption and trade balance data from FRED, Federal Reserve Bank of St. Louis. We detrend the consumption and output series and we focus on the period between 1993q1 and 2012q4.

higher than that of the rich and the ratio is 1.90, which is close to its data counterpart. Incomplete assets together with income shocks are key for this result. In our model income inequality shocks amplify the effect of output shocks particularly on the poor households' endowment. Since there are no other assets that the households can use to insure against these shocks, poor households have higher volatility of consumption than rich households.

Table 10: Business Cycle Statistics Data: Argentina vs. Mexico

	Argentina Data	Mexico Data
corr(spread, y)	-0.88	-0.52
corr(spread, tb)	0.70	0.68
corr(spread, tc)	-0.89	-0.53
corr(tb, y)	-0.64	-0.87
corr(tc, y)	0.98	0.97
std(tc)/std(y)	1.10	1.09
std(c_{poor})/std(c_{rich})	-	1.10
Targets		
Default probability	3%	3%
Debt service-to-GDP	5.45%	4.5%
Mean spread	6.23	4.20

Total consumption and trade balance are denoted by tc and tb, respectively. The consumption volatilities of the rich and poor are yearly, the rest of the statistics are at quarterly frequency.

6 Progressive Income Taxes

In the previous sections, we assume that government distributes the proceeds of the debt payments equally between the households. As mentioned above, these proceeds can function as taxes when they are negative and they can function as transfers, otherwise. Since these payments are lump sum, the burden (benefit) of taxes (transfers) relative to endowment is quite different across the households. Particularly, the burden of lump sum taxes is on the poor. Therefore, this brings up the question: How would the probability of default change in Argentina, if the Argentine economy had a tax system that is as progressive as the advanced economies, such as the U.S. or Germany?

We impose the following tax regime:

$$T(y^i) = \begin{cases} 0 & B - qB' \geq 0, \\ y^i - \lambda (y^i)^{1-\tau} & B - qB' < 0. \end{cases}$$

As τ increases the tax function becomes more progressive, and when $\tau = 1$, both types of households consume equally. The parameter λ is called the shift parameter and determines the average tax rate. If $B - qB'$ is positive, the government only distributes the proceeds of the debt operations across households as transfers similar to the benchmark model. If $B - qB'$ is negative, then the government uses the revenues from the taxes to finance the debt. The budget constraint of the government for the latter case is given as:

$$T(y^1) + T(y^2) + B - qB' = 0. \tag{18}$$

One can solve for λ using the budget constraint of the government:

$$\begin{aligned} y^1 - \lambda (y^1)^{1-\tau} + y^2 - \lambda (y^2)^{1-\tau} + B - qB' &= 0 \\ y - \lambda [(y^1)^{1-\tau} + (y^2)^{1-\tau}] + B - qB' &= 0 \\ \lambda [(y^1)^{1-\tau} + (y^2)^{1-\tau}] &= y + B - qB' \\ \lambda &= \frac{y + B - qB'}{(y^1)^{1-\tau} + (y^2)^{1-\tau}}. \end{aligned}$$

The disposable incomes are denoted by $\tilde{y}^i$ for each type of household i. When $B - qB' < 0$, we get:

$$
\begin{aligned}
\tilde{y}^1 &= \lambda (y^1)^{1-\tau}, \\
&= \frac{(y + B - qB')(y^1)^{1-\tau}}{(y^1)^{1-\tau} + (y^2)^{1-\tau}}. \\
\tilde{y}^2 &= \lambda (y^2)^{1-\tau}, \\
&= \frac{(y + B - qB')(y^2)^{1-\tau}}{(y^1)^{1-\tau} + (y^2)^{1-\tau}}.
\end{aligned}
$$

We can write the budget constraints of the households if the government does not choose to default as:

$$
c^1 = \begin{cases} y^1 + \frac{B - qB'}{2} & B - qB' > 0, \\ \frac{(y+B-qB')(y^1)^{1-\tau}}{(y^1)^{1-\tau}+(y^2)^{1-\tau}} & B - qB' \leq 0. \end{cases}
$$

$$
c^2 = \begin{cases} y^2 + \frac{B - qB'}{2} & B - qB' > 0, \\ \frac{(y+B-qB')(y^2)^{1-\tau}}{(y^1)^{1-\tau}+(y^2)^{1-\tau}} & B - qB' \leq 0. \end{cases}
$$

If the government chooses to default, we assume that the progressive taxes are in effect. The budget constraints during autarky are:

$$
\begin{aligned}
c^1 &= \frac{y^d (y^{d,1})^{1-\tau}}{(y^{d,1})^{1-\tau} + (y^{d,2})^{1-\tau}}, \\
c^2 &= \frac{y^d (y^{d,2})^{1-\tau}}{(y^{d,1})^{1-\tau} + (y^{d,2})^{1-\tau}}.
\end{aligned}
$$

Table 11 shows the average income share for each decile for the given period that we compute using World Bank data. Using the income distribution data from the World Bank between the years 1993-2001 and we compute the before-tax Gini index as 0.47. Using the SWIID data, we find the average after-tax Gini for the same period as 0.44. Based on the income distribution data, we find the value of τ that generates the corresponding after-tax

Gini index as 0.085. We recalibrate the model in order to match 3 percent default probability and mean spread of 6.3, when $\tau = 0.085$[15]. In order to see the effect of progressivity on the default risk, we change the value of $\tau \in \{0.232, 0.509\}$, which are the pertaining values for the U.S. and Germany, respectively.[16]

Table 11: Average income share by deciles between 1993-2001

lowest	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}	9^{th}	highest
0.011	0.026	0.036	0.046	0.0613	0.071	0.096	0.119	0.172	0.364

Table 12: Effect of progressivity on default risk, mean spread and debt-to-GDP

Countries	Argentina ($\tau = 0.085$)	United States ($\tau = 0.232$)	Germany ($\tau = 0.509$)
Default Rate (%)	3.2	2.5	1.0
Mean Spread (%)	4.4	3.3	1.4
Mean Debt-to-GDP ratio (%)	34.4	40.9	52.8

Table 12 shows the effect of tax progressivity on default risk, mean spread and debt-to-GDP ratio. Moving from $\tau = 0.085$ to $\tau = 0.232$, the probability of default decreases from 3.2 percent to 2.5 percent. Moving from $\tau = 0.085$ to $\tau = 0.509$, the probability of default decreases to 1.0 percent. As the default risk declines, the economy generates lower spread and more debt is accumulated. The reason is that taxes reduce the dispersion in marginal utilities of consumption between households, by taxing the rich more than the poor. As the dispersion gets smaller, the government has less incentive to default. Therefore, foreign lenders lend higher levels of debt to the government and the mean spread declines monotonically.

7 Conclusion

This paper studies the role of increasing income inequality in sovereign borrowing and default decisions using a stochastic general equilibrium model in a small open economy with

[15] The calibrated parameters are $\beta = 0.87$, $d_0 = -0.63$ and we fix $d_1 = 0.095$.

[16] We obtain the value for the progressivity parameter for the U.S. from Heathcote et al. (2014) and the value for Germany from Chang et al. (2016).

endogenous default risk. To motivate the idea, we analyze the nexus among the Gini index, sovereign bond ratings and GDP per capita using a panel data set. The results show that high inequality lowers the creditworthiness of long-term government bonds significantly. Next, using a model that belongs to the class of models of Eaton and Gersovitz (1981) and extending it to allow for heterogeneous agents and shocks to the distribution of income, the paper shows analytically that inequality shocks can generate a high probability of default when the markets are incomplete. Using Argentine data, the model predicts a default probability of 2.8 percent and also matches the business-cycle characteristics observed in the data, such as the high volatility of consumption and output, the counter-cyclical interest rates, and positive correlations between the trade balance and interest rates, as well as, inequality and interest rates. Our model's contribution is to highlight the redistributive effects of default as a policy that improves the welfare of the households. The model can also explain the differences in consumption volatilities across different income groups, which has not been shown by the earlier papers in the literature. As a policy extension, we show that progressive income taxes can reduce the default risk and increase the debt-to-output ratio.

Rising income inequality is a general problem that many countries have experienced. Therefore, it is important to understand how inequality induces economic crises and sovereign defaults that last several years and cause large losses. Even though our paper provides a first step toward analyzing the role of income inequality, we abstract from the determinants of income inequality and model it as an exogenous shock. We think that it is also important to study what drives high income inequality and how it affects agents' welfare and a government's decision to default. We leave these issues for future study.

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Appendix

7.1 Tables

Table 14: VAR estimations for different countries

Country	ρ_{yy}	$\rho_{y\gamma}$	$\rho_{\gamma y}$	$\rho_{\gamma\gamma}$	σ^2_y	$\sigma_{y\gamma}$	$\sigma_{\gamma y}$	σ^2_γ
Brazil	0.34	-0.25	0.09	0.64	5.6x10^{-4}	5x10^{-5}	5x10^{-5}	8x10^{-5}
Colombia	0.44	0.09	-0.15	0.33	1.4x10^{-4}	2x10^{-5}	2x10^{-5}	8x10^{-5}
Costa Rica	0.33	-0.07	0.05	0.74	4.5x10^{-5}	-1x10^{-5}	-1x10^{-5}	9x10^{-5}
Dominican Republic	0.26	-0.33	0.07	0.71	6x10^{-4}	-1.3x10^{-5}	-1.3x10^{-5}	9x10^{-5}
Ecuador	0.01	-0.33	0.20	0.82	1.3x10^{-3}	-1.8x10^{-4}	-1.8x10^{-4}	2.3x10^{-4}
Paraguay	-0.74	0.24	-0.05	0.73	4x10^{-4}	-4x10^{-5}	-4x10^{-5}	7x10^{-5}
Uruguay	0.26	-0.33	0.07	0.71	6x10^{-4}	-1.3x10^{-5}	-1.3x10^{-5}	9x10^{-5}
Argentina	0.28	-0.56	0.05	0.79	1.2x10^{-3}	-2x10^{-4}	-2x10^{-4}	1.3x10^{-4}

In this VAR analysis, we assume that log output and the inequality follow a VAR(1) process such that

$$\begin{bmatrix} log(y_t) \\ \gamma_t \end{bmatrix} = \begin{bmatrix} c_y \\ c_\gamma \end{bmatrix} + \begin{bmatrix} \rho_{yy} & \rho_{y\gamma} \\ \rho_{\gamma y} & \rho_{\gamma\gamma} \end{bmatrix} \begin{bmatrix} log(y_{t-1}) \\ \gamma_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{\gamma t} \end{bmatrix}$$

where

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{\gamma t} \end{bmatrix}$$

$$E[\boldsymbol{\varepsilon}] = \mathbf{0} \quad \text{and} \quad Var[\boldsymbol{\varepsilon}] = \begin{bmatrix} \sigma^2_y & \sigma_{y\gamma} \\ \sigma_{\gamma y} & \sigma^2_\gamma \end{bmatrix}$$

Table 13: Fitch Credit Rating Conversion Table

Fitch Rating	Score
AAA	23
AA+	22
AA	21
AA-	20
A+	19
A	18
A-	17
BBB+	16
BBB	15
BBB-	14
BB+	13
BB	12
BB-	11
B+	10
B	9
B-	8
CCC+	7
CCC	6
CCC-	5
CC	4
C	3
DDD	2
D	1
RD	1

7.2 Proofs of Propositions

7.2.1 Proof of Proposition 1

The proof is similar to Arellano (2008).

First we show that value of repayment is increasing i asset holdings. For all $\{y, \gamma\} \in D(B^2)$,

$$\begin{aligned} \frac{y(1-\gamma)}{2} + \frac{B^2 - q(B', y.\gamma)B'}{2} &> \frac{y(1-\gamma)}{2} + \frac{B^1 - q(B', y, \gamma)B'}{2}, \\ \frac{y(1+\gamma)}{2} + \frac{B^2 - q(B', y.\gamma)B'}{2} &> \frac{y(1+\gamma)}{2} + \frac{B^1 - q(B', y, \gamma)B'}{2}. \end{aligned}$$

So,

$$\begin{aligned} u\left(\frac{y(1-\gamma)}{2} + \frac{B^2 - q(B', y.\gamma)B'}{2}\right) &+ u\left(\frac{y(1+\gamma)}{2} + \frac{B^2 - q(B', y.\gamma)B'}{2}\right) + \beta E v^o(B', y', \gamma') \geq \\ u\left(\frac{y(1-\gamma)}{2} + \frac{B^1 - q(B', y.\gamma)B'}{2}\right) &+ u\left(\frac{y(1+\gamma)}{2} + \frac{B^1 - q(B', y.\gamma)B'}{2}\right) + \beta E v^o(B', y', \gamma'). \end{aligned}$$

Therefore, for all $\{y, \gamma\} \in D(B^2)$,

$$\begin{aligned} &u\left(\frac{y(1-\gamma)}{2}\right) + u\left(\frac{y(1+\gamma)}{2}\right) + \beta E[\theta v^o(0, y', \gamma') + (1-\theta)v^d(y', \gamma')] > \\ &u\left(\frac{y(1-\gamma)}{2} + \frac{B^2 - q(B', y.\gamma)B'}{2}\right) + u\left(\frac{y(1+\gamma)}{2} + \frac{B^2 - q(B', y.\gamma)B'}{2}\right) + \beta E v^o(B', y', \gamma') \geq \\ &u\left(\frac{y(1-\gamma)}{2} + \frac{B^1 - q(B', y.\gamma)B'}{2}\right) + u\left(\frac{y(1+\gamma)}{2} + \frac{B^1 - q(B', y.\gamma)B'}{2}\right) + \beta E v^o(B', y', \gamma'). \end{aligned}$$

Hence, any pair of $\{y, \gamma\}$ that is in $D(B^2)$, we have $\{y, \gamma\} \in D(B^1)$.
Let $d(B, y', \gamma')$ denote the equilibrium default decision rule. Default probability satisfies

$$\delta(B, y', \gamma') = \int d(B, y', \gamma') f((y', \gamma'), y, \gamma) d(y', \gamma')$$

Since any $\{y, \gamma\} \in D(B^2)$, we have $D(B^2) \subseteq D(B^1)$, if $d(B^2, y', \gamma') = 1$, then $d(B^1, y', \gamma') = 1$. Hence, $\delta(B^1, y, \gamma) \geq \delta(B^2, y, \gamma)$. □

7.2.2 Proof of Proposition 2

The bond price is defined as $q(B', y, \gamma) = \frac{1-\delta(B', y, \gamma)}{1+r}$. Using Proposition 1, we have $B^1 < B^2 \leq 0$ and $\delta(B^2, y, \gamma) < \delta(B^1, y, \gamma)$. Hence, we get $q(B^2, y, \gamma) > q(B^1, y, \gamma)$. $\square$

7.3 Solution Algorithm

To solve the model numerically, we use the discrete state-space method. We discretize the asset space using a finite set of grid points, making sure that the minimum and the maximum points on the grid do not bind when we compute the optimal debt decision. Our solution algorithm for the benchmark model is the following:[17]

1. Guess that the initial price is the reciprocal of the risk-free interest rate, and the initial value function is equal to the autarky value.

2. Given a price $q(B', y, \gamma)$ and $v^o(B, y, \gamma)$, solve for the optimal policy functions and update the value of option given as equation (10) by comparing $v^c(B, y, \gamma)$ and $v^d(y, \gamma)$.

3. Given the price function, compute the default probabilities.

4. Update the price function using equation (9).

5. We simultaneously check whether the initial guesses for price and the value of option are close enough to their updated values. If not, we update the initial values and iterate steps 2-4 until both bond price and the value of option functions converge.

[17] We use the same algorithm to solve the models with a single type of shock. For instance, for Model II, the price function is denoted as $q(B', y)$, and value of option for default or repayment is denoted as $v^o(B, y)$.